Question 1

A linear translation stage (horizontal) with a ball screw needs to be driven by a stepper motor to carry a load of 200kg. The stage is on linear guides/bearings that have a coefficient of friction $\mu_{linear-bearing}$ = 0.025. The ball screw has pitch P of 2mm/revolution, weight of 3kg and efficiency η of 85%. The stage has a travel of 300mm and needs to cover this distance in 20 seconds. Assume the inertia of any couplings are negligible.

A) The whole system should have an accuracy of 100 microns. Calculate the step increment using the stepper motor full step size, comment on whether the system is can achieve this and if not what changes you would make to the system design.

B) Select an encoder for the application, specify where to place it and calculate the measurement resolution of the system.

C) Calculate the total torque required for the application and select a stepper motor from the ESS Components Standard. The motor will be run with the Beckhoff EL7041-0052 stepper drive.

Part A)

Determine the smallest full step increment that the load can theoretically make

$$Resoloution_{full-step} = \frac{P_{screw}}{MotorSteps}$$

$$Resolution_{full-step} = \frac{0.002\ mm}{200} = 1 \times 10^{-5}\ m$$

$$Resolution_{full-step} = 10\ \mu m$$

Is 10 μm positioning good enough for the 100 μm requirement? It's on the limit but with micro-stepping it's probably acceptable.

Part B)

From the ESS Standard the first choice for encoder is the Baumer BMMV 30S1G SSI encoder which has 16 bit single turn.

$$Resolution = \frac{P}{2^{Bits}} = \frac{2\ mm}{2^{16}\ bits}$$

$$Resolution = 0.031\ \mu m$$

This is measurement resolution is adequate for the requirements.

Part C)

Total torque

To move the load from stationary

$$\tau_{total} = \tau_{load} + \tau_{accel}$$

$$\tau_{load} = \tau_{friction} + \tau_{gravity} \quad \& \quad \tau_{accel} = J_{toral} \times \omega$$

In this case $\tau_{gravity} = 0$

Load torque

Calculate axial force of load on the stage

$$F_{axial} = m \times g \times \mu_{linear-bearing}$$

$$F_{axial} = 200\ kg \times 9.8\ m/s^2 \times 0.02$$

$$F_{axial} = 39.2\ kg \cdot m/s^2 (N)$$

Calculate torque required to move axial load on the rails

$$\tau_{load} = \frac{F_A \times P_{screw}}{2 \times \pi \times \eta}$$

$$\tau_{load} = \frac{39.2\ N \times 0.002\ m}{2 \times \pi \times 0.85}$$

$$\boldsymbol{\tau_{load} = 0.015\ Nm}$$

Select motor/gearbox for first iteration

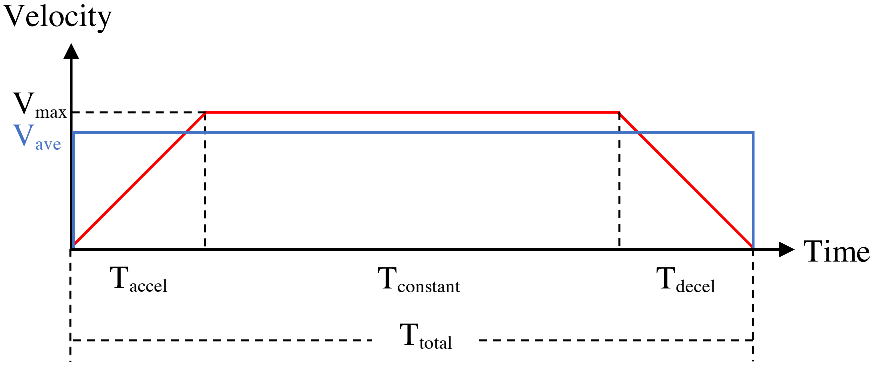
Using a margin factor of at least double the estimate torque required would be:

$$\tau = 2\ \times\ 0.015\ Nm = 0.03\ Nm$$

Note: the torque requirement to accelerate the load still needs to be added but a motor inertia value is required to calculate this. Therefore a best guess should be made to choose a motor for the first iteration. Looking at the motors in the ESS standard starting from the lowest torque, the Phytron ZSS 33.200.1.2, can provide ~0.05 Nm at V_{Max} which is suitable the first iteration, keeping in mind the torque required to accelerate still needs to be added.

Create a motion profile

Assume a trapezoidal profile as a first approach



Choose an acceleration of 1.5 seconds to start with.

$$T_{constant} = T_{total} - 2 \times T_{accel} = 20 - 1.5 \times 2 = 17$$

The total distance traveled is the sum of the areas of the three segments.

$$Distance_{total} = D_{accel} + D_{constant} + D_{decel}$$

Insert formulas for triangle, rectangle and triangle.

$$D_{total} = \left(\frac{b \times h}{2}\right) + (b \times h) + \left(\frac{b \times h}{2}\right)$$

$$D_{total} = \left(\frac{T_{accel} \times V_{max}}{2}\right) + (T_{const} \times V_{max}) + \left(\frac{T_{decel} \times V_{max}}{2}\right)$$

Rearrange to find max velocity

$$V_{max} = \frac{D_{total}}{\left(\frac{T_{accel} + T_{decel}}{2}\right) + T_{const}}$$

$$V_{max} = \frac{300mm}{\left(\frac{1.5 + 1.5}{2}\right) + 17}$$

$$V_{max} = 16.2\ mm/s$$

Convert to motor rpm to later to look at torque speed curve

$$V_{max} = \frac{16.2\,mm/s}{2mm} \times 60\,sec = 486\,rpm$$

Using the velocity and acceleration time calculate maximum acceleration of the stage

$$A_{max} = \frac{16.2\,mm/s}{1.5\,s} = 10.8\ mm/s^2$$

Convert to motor acceleration

$$\omega_{max} = \frac{10.8\ mm/s^2}{2\,mm/rev} \times 2\pi = 34\,rad/s^2$$

System inertia

$$J_{total} = J_{motor} + J_{gearbox} + (J_{coupling} + J_{lead-screw} + J_{load})/GR^2$$

Using the proposed motor. Note: the moment of inertia of the motor is chosen as estimation and can be changed on a second iteration.

$$J_{motor} = 0.018\,kg \cdot cm^2 = 1.8 \times 10^{-6}\,kg \cdot m^2$$

Inertia of screw (solid cylinder)

$$J_{lead-screw} = \frac{m \cdot r^2}{2} = \frac{3\,kg \cdot (0.025\,m)^2}{2} = 9.4 \times 10^{-4}\,kg \cdot m^2$$

Inertia of screw-driven load (screw's lead must be taken into account)

$$J_{load} = m \cdot \left(\frac{P_{screw}}{2\pi}\right)^2 = 190\,kg \cdot \left(\frac{0.002\,m}{2\pi}\right)^2 = 2 \times 10^{-5}\,kg \cdot m^2$$

Total system inertia

$$J_{total} = 1.8 \times 10^{-6}\,kg \cdot m^2 + 9.4 \times 10^{-4}\,kg \cdot m^2 + 2 \times 10^{-5}\,kg \cdot m^2$$

$$J_{total} = 9.6 \times 10^{-4}\,kg \cdot m^2$$

Accelerating Torque

Calculate the torque required to accelerate the system:

$$\tau_{accel} = J_{Toral} \times \omega_{Max}$$

$$\tau_{accel} = 6.5 \times 10^{-4}\,kg \cdot m^2 \times 34\,rad/s^2$$

$$\boldsymbol{\tau_{accel} = 0.047\,Nm}$$

Total Torque

Calculate the torque by adding load torque and acceleration torque

$$\tau_{total} = \tau_{load} + \tau_{accel}$$

$$\tau_{total} = 0.015\,Nm + 0.033\,Nm$$

$$\boldsymbol{\tau_{total} = 0.047\,Nm}$$

Multiplying by a margin factor of 2

$$\therefore \tau_{required} = 0.095\,mNm$$

At $\sim 486\ rpm$ the ZSS 33.200.1.2 can give $\sim 0.05\ Nm$. This does not give us enough safety margin so the next size of motor should be taken. The ZSS 43.200.2.5 can provide $\sim 0.185\ Nm$ at $\sim 486\ rpm$ so this is a safer choice for this application.

Inertia Matching (required?)

$$J_{ratio} = \frac{J_{system-load}}{J_{motor}} = \frac{9.6 \times 10^{-4}\ kg \cdot m^2}{1.8 \times 10^{-6}\ kg \cdot m^2} = 531$$

It is best to keep the load-to-motor inertia ratio below 10. To achieve better ratio we could introduce a 10:1 gearbox (or select a bigger motor) and thus reduce the ratio. If we used a 10:1 gearbox.

$$J_{system-Load} = J_{gearbox} + (J_{lead-screw} + J_{load})/GR^2$$

$$J_{system-load} = \frac{(9.4 \times 10^{-4}\ kg \cdot m^2 + 1.93 \times 10^{-5}\ kg \cdot m^2)}{10^2}$$

$$J_{system-load} = 9.6 \times 10^{-4}\ kg \cdot m^2$$

$$J_{ratio} = \frac{J_{system-load}}{J_{motor}} = \frac{9.6 \times 10^{-4}\ kg \cdot m^2}{1.8 \times 10^{-6}\ kg \cdot m^2} = 5.3$$

If the next size motor is selected

$$J_{ratio} = \frac{J_{system-load}}{J_{motor}} = \frac{9.6 \times 10^{-4}\ kg \cdot m^2}{7.7 \times 10^{-6}\ kg \cdot m^2} = 125$$

This might be an adequate option if dynamics aren't too important.

Question 2

Use the same parameters from the previous question, this time perform the design for vertical stage carrying a detector in a vacuum tank. However for this application the time needed to travel the distance has been relaxed to 4 minutes and the weight is increased to 250kg.

A) The whole system should have an accuracy of 25 microns. Calculate the step increment using the stepper motor full step size, comment on whether the system is can achieve this and if not what changes you would implement. Refer to the data sheets if needed.

B) Select an encoder for the application, specify where to place it and calculate the measurement resolution of the system.

C) Calculate the total torque required to accelerate the load vertically and select a stepper motor/gearbox from the ESS Components Standard. The motor will be run with the Beckhoff EL7041-0052 stepper drive.

D) Is anything required to prevent the system back driving?

Part A)

Determine the smallest full step increment that the load can make

$$R_{full-step} = \frac{P_{screw}}{MotorSteps}$$

$$Resolution_{full-step} = \frac{0.002\ mm}{200} = 1 \times 10^{-5}\ m$$

$$Resolution_{full-step} = 10\ \mu m$$

Is 10 μm positioning good enough for the 25 μm requirement? Not in this case so, require at least 10:1 so add a gearbox. Assume a 10:1 gearbox as a first approximation.

$$Resolution_{full-step} = \frac{0.002\ mm}{200 \times 10} = 1 \times 10^{-6}\ m$$

$$Resolution_{full-step} = 1\ \mu m$$

Part B)

Looking at the ESS Standard the first choice for encoder is the Baumer BMMV 30S1G SSI encoder which has 16 bit single turn.

$$Resolution = \frac{P}{2^{Bits}} = \frac{2\ mm}{2^{16}\ bits}$$

$$Resolution = 0.031\ \mu m$$

This is measurement resolution is adequate for the requirements.

Part C)

Total torque

To move the load from stationary

$$\tau_{total} = \tau_{load} + \tau_{accel}$$

$$\tau_{load} = \tau_{friction} + \tau_{gravity} \quad \& \quad \tau_{accel} = J_{toral} \times \omega$$

Load torque

Calculate axial force of load

$$F_{axial} = m \times g$$

$$F_{axial} = 200\ kg \times 9.8\ m/s^2$$

$$F_{axial} = 2450\ kg \cdot m/s^2 (N)$$

Calculate torque required to move axial load on the rails

$$\tau_{load} = \frac{F_A \times P_{screw}}{2 \times \pi \times \eta \times GR}$$

$$\tau_{load} = \frac{2450\ N \times 0.002\ m}{2 \times \pi \times 0.85 \times 10}$$

$$\tau_{load} = 0.092\ Nm$$

Select motor/gearbox for first iteration

Using a margin factor of at least double the estimate torque required would be:

$$\tau = 2\ \times\ 0.092\ Nm = 0.183\ Nm$$

Note: the torque requirement to accelerate the load still needs to be added but a motor inertia value is required to calculate this. Therefore a best guess should be made to choose a motor for the first iteration. Looking at the motors in the ESS standard for harsh environments starting from the lowest torque, the Phytron VSS 43.200.2.5, can provide ~0.16 Nm at V_{Max}. Remembering that the torque due to acceleration still needs to be added, this is probably not a sufficient margin. The next motor in the list is the VSS 57.200.2.5, which can provide ~1.2 Nm and will be a safer choice. Using this for the first iteration and looking at the gearboxes available for this motor, there are ratios of 6.25 or 12.08 closest to 10. Take 12.08 for this iteration.

Recalculate torque required using this gearbox

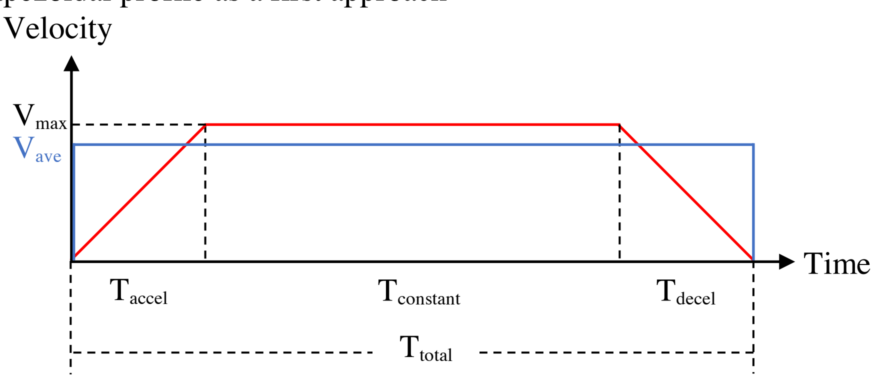
$$\tau_{load} = \frac{F_A \times P_{screw}}{2 \times \pi \times \eta \times GR}$$

$$\tau_{load} = \frac{2450\ N \times 0.002\ m}{2 \times \pi \times 0.85 \times 12.08}$$

$$\boldsymbol{\tau_{load} = 0.089\ Nm}$$

Create a motion profile

Assume a trapezoidal profile as a first approach



Choose an acceleration of 1.5 seconds to start with.

$$T_{constant} = T_{total} - 2 \times T_{accel} = 240 - 1.5 \times 2 = 237$$

The total distance traveled is the sum of the areas of the three segments.

$$Distance_{total} = D_{accel} + D_{constant} + D_{decel}$$

Insert formulas for triangle, rectangle and triangle.

$$D_{total} = \left(\frac{b \times h}{2}\right) + (b \times h) + \left(\frac{b \times h}{2}\right)$$

$$D_{total} = \left(\frac{T_{accel} \times V_{max}}{2}\right) + (T_{const} \times V_{max}) + \left(\frac{T_{decel} \times V_{max}}{2}\right)$$

Rearrange to find max velocity

$$V_{max} = \frac{D_{total}}{\left(\frac{T_{accel} + T_{decel}}{2}\right) + T_{const}}$$

$$V_{max} = \frac{300mm}{\left(\frac{1.5 + 1.5}{2}\right) + 237}$$

$$V_{max} = 1.3\, mm/s$$

Convert to motor rpm to later to look at torque speed curve (required later)

$$V_{max} = \frac{1.3\, mm/s}{2mm} \times 60\, sec = 455\, rpm$$

Use the velocity and acceleration time calculate the max acceleration of the stage (mm/s^2)

$$A_{max} = \frac{1.3\, mm/s}{1.5\, s} = 0.83\ mm/s^2$$

Convert to motor acceleration (rad/s^2)

$$\omega_{max} = \frac{0.83\ mm/s^2}{2\, mm/rev} \times 2\pi = 31.8\, rad/s^2$$

System inertia

$$J_{total} = J_{motor} + J_{gearbox} + \left(J_{coupling} + J_{lead-screw} + J_{load}\right)/GR^2$$

Using the proposed motor. Note: the moment of inertia of the motor is chosen as estimation and can be changed on a second iteration.

$$J_{motor} = 0.24\, kg \cdot cm^2 = 2.4 \times 10^{-6}\, kg \cdot m^2$$

Using the proposed gearbox

$$J_{gearbox} = 0.55\, kg \cdot cm^2 = 5.5 \times 10^{-6}\, kg \cdot m^2$$

Inertia of screw (solid cylinder)

$$J_{lead-screw} = \frac{m \cdot r^2}{2} = \frac{3\, kg \cdot (0.025\, m)^2}{2} = 9.4 \times 10^{-4}\, kg \cdot m^2$$

Inertia of screw-driven load (screw's lead must be taken into account)

$$J_{load} = m \cdot \left(\frac{P_{screw}}{2\pi}\right)^2 = 200\, kg \cdot \left(\frac{0.002\, m}{2\pi}\right)^2 = 2 \times 10^{-5}\, kg \cdot m^2$$

Total system inertia

$$J_{total} = 2.4 \times 10^{-5}\ kg \cdot m^2 + 5.5 \times 10^{-6}\ kg \cdot m^2 + (9.4 \times 10^{-4}\ kg \cdot m^2 + 2 \times 10^{-5}\ kg \cdot m^2)/12.08^2$$

$$J_{total} = 3.6 \times 10^{-5}\ kg \cdot m^2$$

Accelerating Torque

Calculate the torque required to accelerate the system:

$$\tau_{accel} = J_{Toral} \times \omega_{Max}$$

$$\tau_{accel} = 3.6 \times 10^{-4}\ kg \cdot m^2 \times 31.8\ rad/s^2$$

$$\boldsymbol{\tau_{accel} = 0.001\ Nm}$$

Total Torque

Calculate the required torque by adding load torque and acceleration torque

$$\tau_{total} = \tau_{load} + \tau_{accel}$$

$$\tau_{total} = 0.089\ Nm + 0.001\ Nm$$

$$\boldsymbol{\tau_{total} = 0.09\ Nm}$$

Multiplying by a margin factor of 2

$$\therefore \tau_{required} = 0.18\ mNm$$

Earlier the VSS 57.200.2.5 was chosen as a first iteration with a ~0.56 Nm at ~455 rpm so this should be sufficient.

Part D)

Back driving

$$\tau_{backdrive} = \frac{F_A \times P_{screw} \times \eta_{screw} \times \eta_{gearbox}}{2 \times \pi \times GR}$$

$$\tau_{backdrive} = \frac{1960 \times 0.002\ m/rev \times 0.85 \times 0.85}{2 \times \pi \times 12.08}$$

$$\tau_{backdrive} = 0.047\ Nm$$

The VSS 57.200.2.5 has a detent torque of 0.05 Nm. Is this adequate to prevent back driving?