

Diffraction-grating VCN interferometry and experimental search for neutron electric charge

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Contents:

1. I will not discuss “why” to measure q_n , just “how” to measure.
2. Earlier attempts and current experimental limit q_n .
3. Interferometrical approach using grating interferometers.
4. VCN grating interferometer in gravitational field, specific requirements .
5. Experiment on neutron charge quest at ESS.

Previous experiments and limits on neutron charge q_n

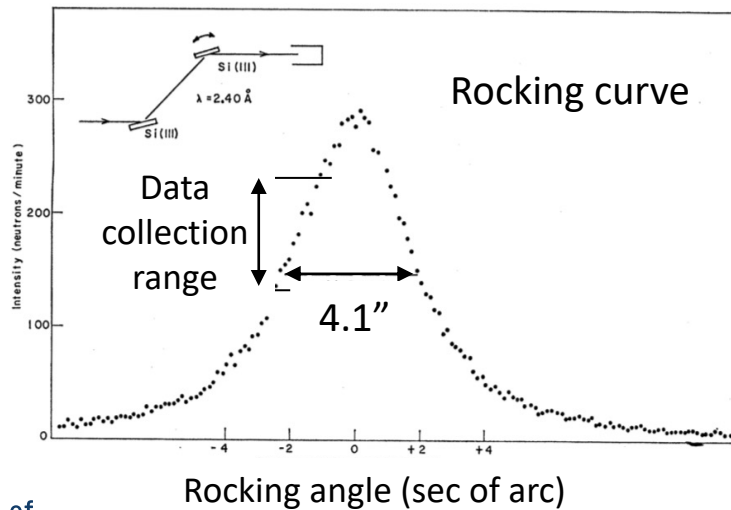
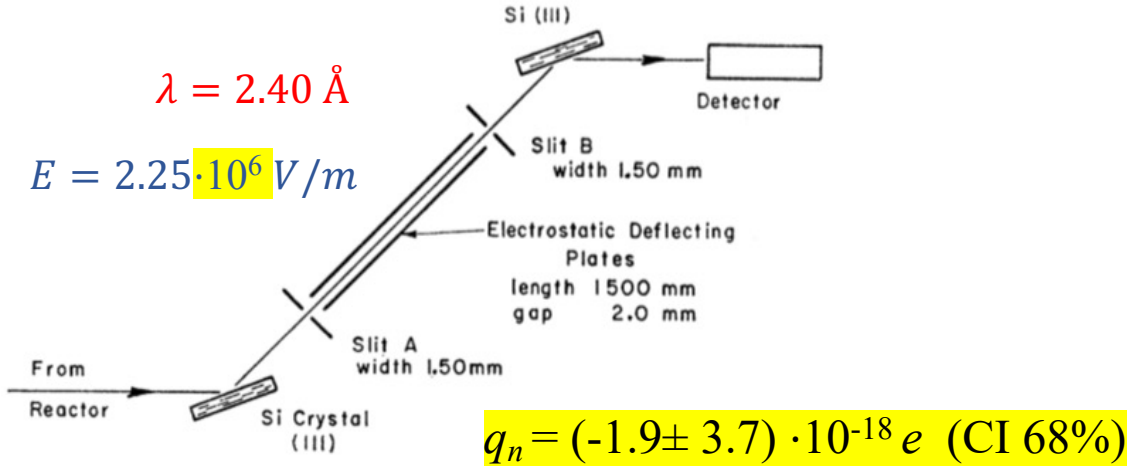
PHYSICAL REVIEW

VOLUME 153, NUMBER 5

25 JANUARY 1967

Experimental Limit for the Neutron Charge*

C. G. SHULL, K. W. BILLMAN, AND F. A. WEDGWOOD†



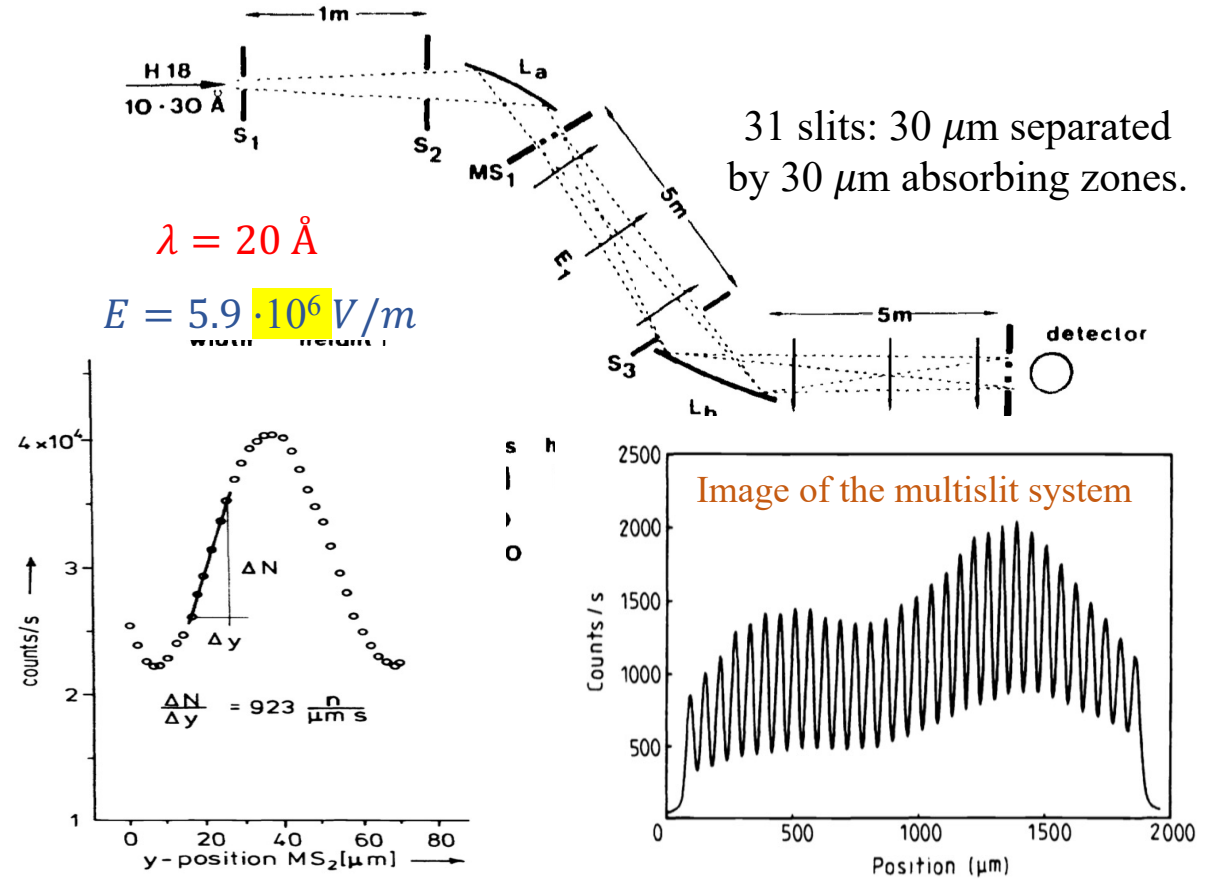
PHYSICAL REVIEW D

VOLUME 25, NUMBER 11

1 JUNE 1982

Experimental limit for the charge of the free neutron

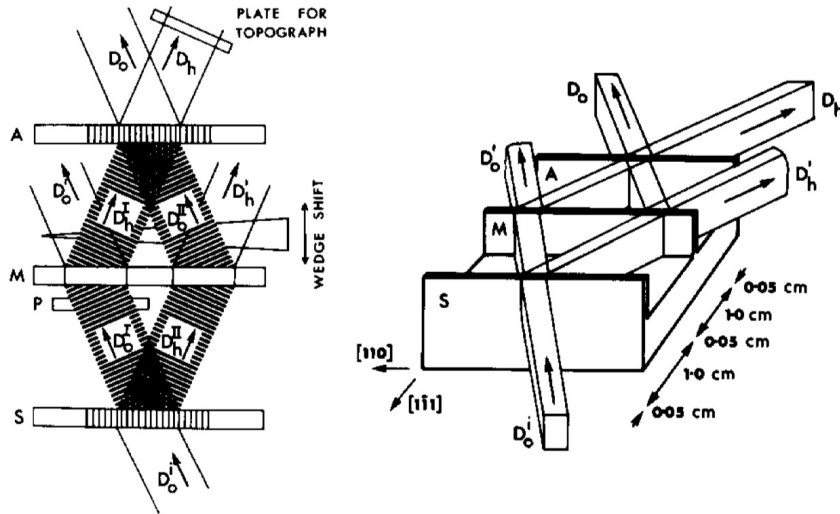
R. Gähler, J. Kalus, W. Mampe



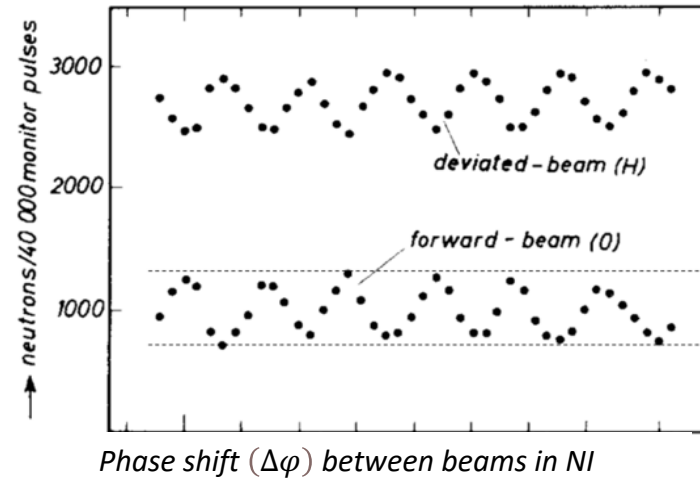
$q_n = (-0.4 \pm 1.1) \cdot 10^{-21} e$ (CI 68%)

Perfect crystal neutron interferometer

Perfect crystal neutron LLL interferometer



Introduced phase shift ($\Delta\varphi$) between beams in the NI

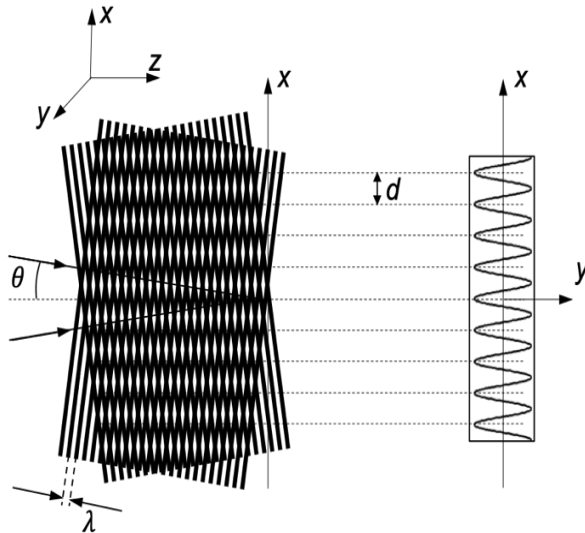


$$I_H(\Delta\varphi) = B - A \cos(\Delta\varphi)$$

Swapping intensity
between O- and H-beams

$$I_O(\Delta\varphi) = V(1 + \cos(\Delta\varphi))$$

H. Rauch et al., 1974



Alternatively:

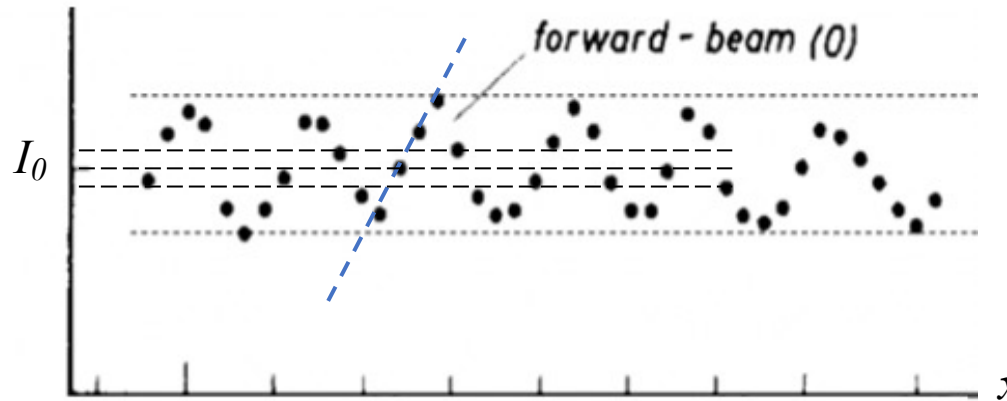
- crossing coherent waves produce interference pattern of period d
- these interference fringes are superimposed with crystal lattice (d) => Moiré effect (fringes)
- $\Delta\varphi$ results in the lateral shift of interference fringes w.r.t. crystal lattice
- this leads to oscillations of Moiré fringes, i.e. oscillations of recorded intensity

Gedanken experiment with crystal interferometer: q_n

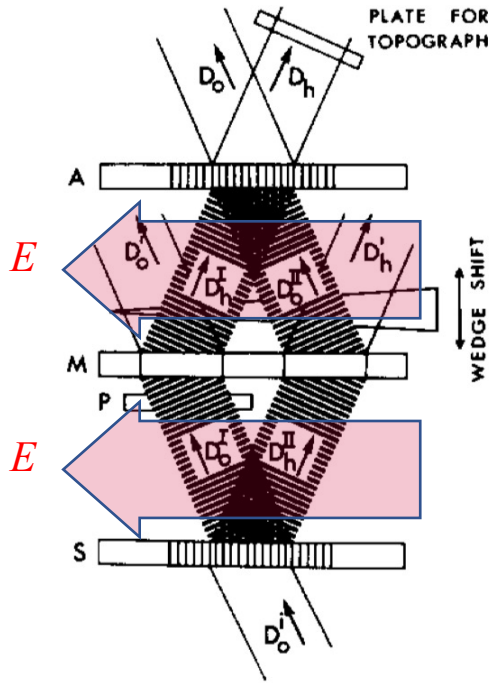
Electric field across neutron beams => shift of interference pattern:

$$\Delta x = \frac{1}{2} q E \left(\frac{L}{v} \right)^2 = \frac{1}{2} q E \left(\frac{L}{h} m \lambda \right)^2 \sim q E L^2 \lambda^2$$

$$I(\Delta x) = I_0 V \left(1 + \cos \frac{2\pi}{d} \Delta x \right) \quad \rightarrow \quad \text{Shift by } d \text{ (lattice spacing) corresponds to full intensity oscillation}$$



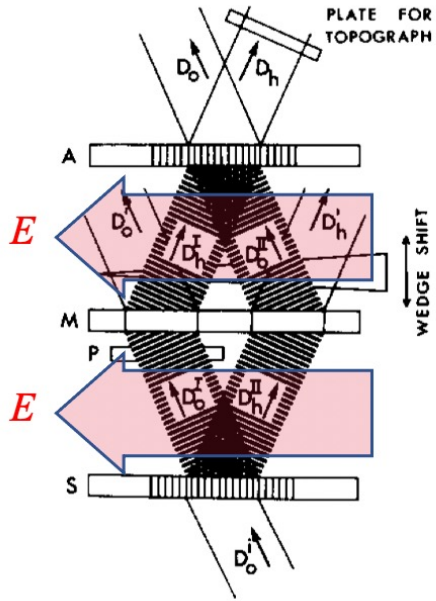
$$q = \frac{\sqrt{2} d}{\pi \sqrt{I_0}} \frac{1.5}{V E L^2 \lambda^2} \left(\frac{h}{m} \right)^2$$



For $E = 60 \text{ kV/cm}$, $L = 5 \text{ cm}$, $\lambda = 2 \text{ \AA}$, $d = 1.92 \text{ \AA}$ $q_n \geq 3 \cdot 10^{-20} e$ (CI 90%) in 100 days

Important: $q \sim \frac{d}{\sqrt{I_0} E L^2 \lambda^2}$ - this is a kind of FOM

Neutron interferometer with larger length and wavelength



$$q_n \sim \frac{d}{\sqrt{I_0} E L^2 \lambda^2}$$

Now imagine we can modify our interferometer towards larger length and wavelength, however with corresponding increase of d .



Scaling to VCN:

$$\lambda: 2 \text{ \AA} \rightarrow 20 \text{ \AA} \Rightarrow \times 10^2$$

$$L: 5 \text{ cm} \rightarrow 5 \text{ m} \Rightarrow \times 10^4$$

X

$$d: 2 \text{ \AA} \rightarrow 1 \text{ \mu m} \Rightarrow \times (2 \cdot 10^{-4})$$

$$I_0: \sqrt{\lambda^{-5}} \Rightarrow \times (3 \cdot 10^{-3})$$

X

Thermal to cold
neutron source: x 15

Total gain about 10 \Rightarrow one can put a harder limit on q_n

However, for cold neutrons one should use other than the Laue diffraction coherent splitting of neutron waves

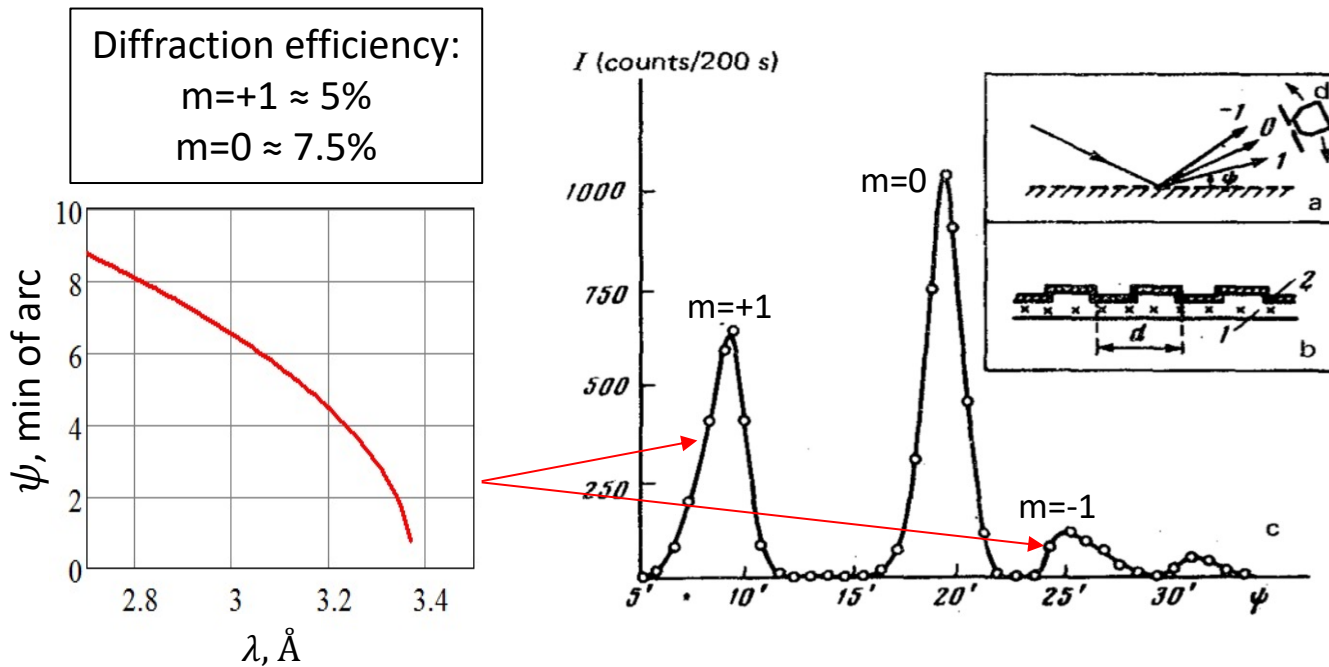
(Very) cold neutrons: coherent beam splitting

For cold neutrons one should employ other than the Laue diffraction coherent splitting of neutron waves: diffraction on periodical structures (gratings) or reflection from semi-transparent coatings.

Effective neutron diffraction gratings: modulated surface relief

A.Ioffe et al, JETP letters 33, 374 (1981)

$d = 21 \mu\text{m}$ $\lambda = 2.7 \text{ \AA}$, $\Delta\lambda/\lambda = 32\%$ (Ni mirror)

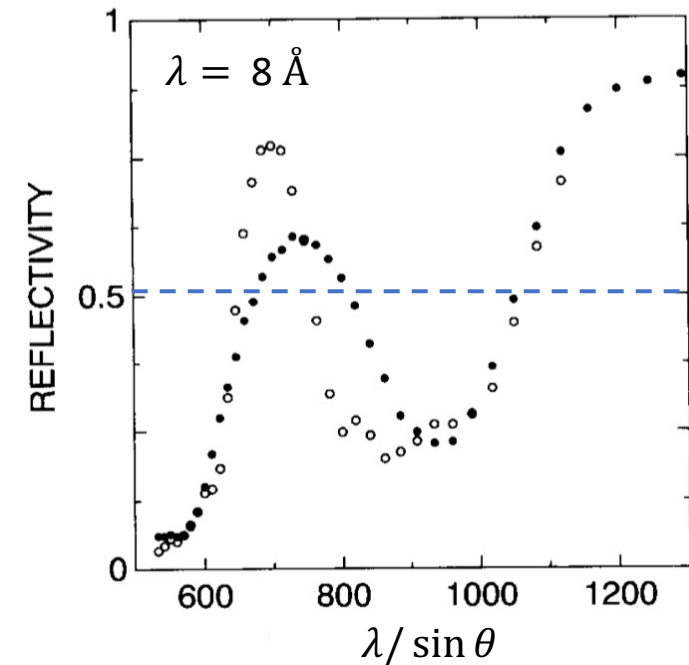


Note asymmetry: spectrum of incident beam => spectroscopy

Coherent beam splitter

T. Ebisawa et al, NIM A 344, 597 (1994)

V-Ti multilayer mirror with spacing $d=360 \text{ \AA}$



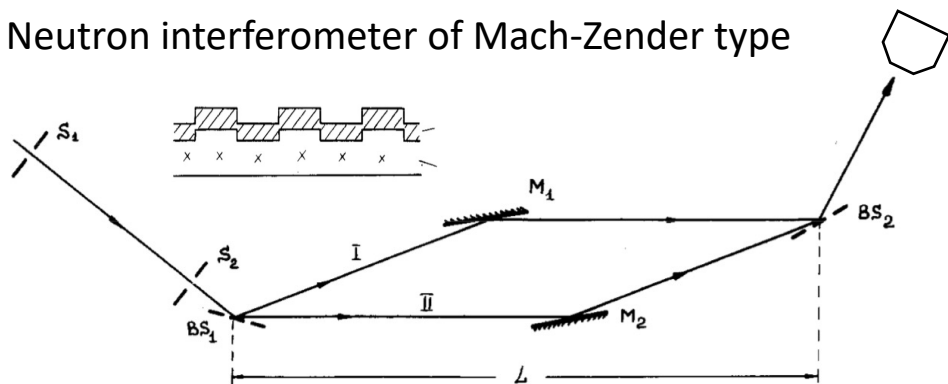
Cold neutron interferometers

Volume 111, number 7 PHYSICS LETTERS 30 September 1985

TEST OF A DIFFRACTION GRATING NEUTRON INTERFEROMETER

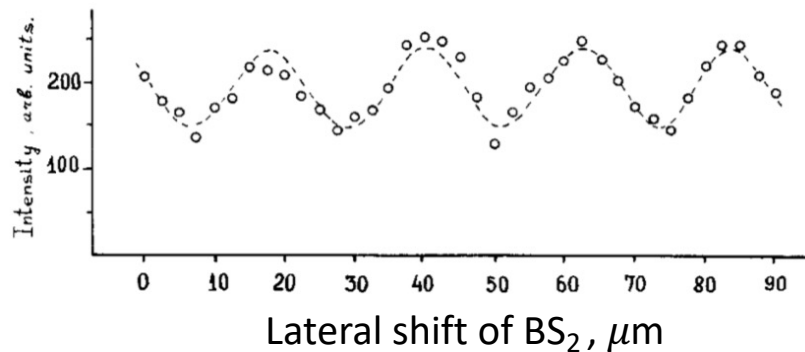
A.I. IOFFE, V.S. ZABIYAKIN and G.M. DRABKIN

Neutron interferometer of Mach-Zender type



Moire pattern with a period d :

$$\lambda = 3.15 \text{ \AA} \quad d = 21 \text{ \mu m}$$



PHYSICAL REVIEW A

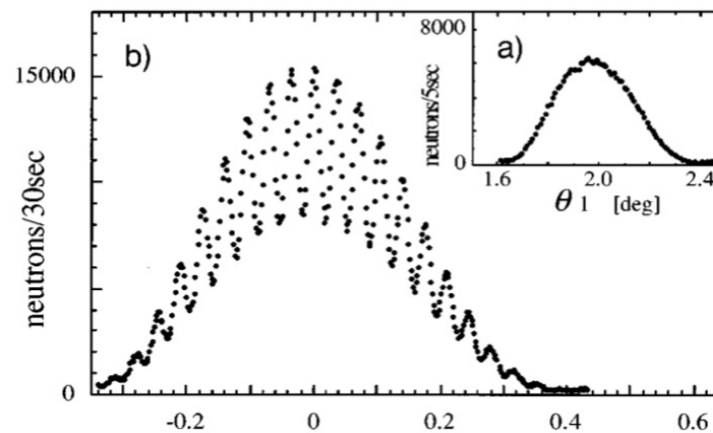
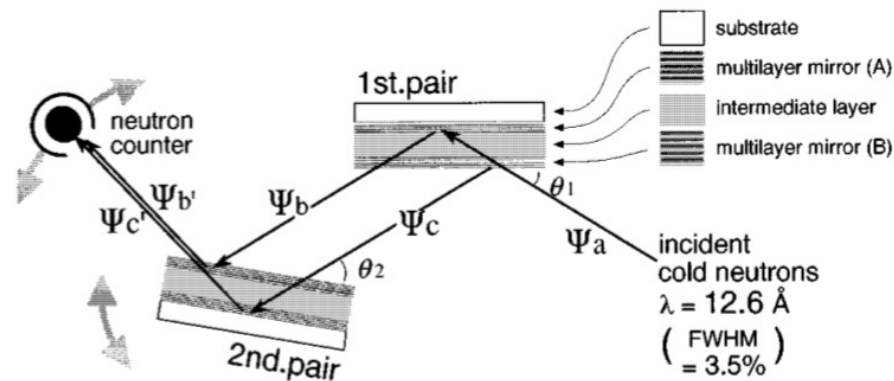
VOLUME 54, NUMBER 1

JULY 1996

Interferometer for cold neutrons using multilayer mirrors

Haruhiko Funahashi,^{1,*} Toru Ebisawa,¹ Tomohito Haseyama,² Masahiro Hino,³ Akira Masaike,² Yoshié Otake,⁴

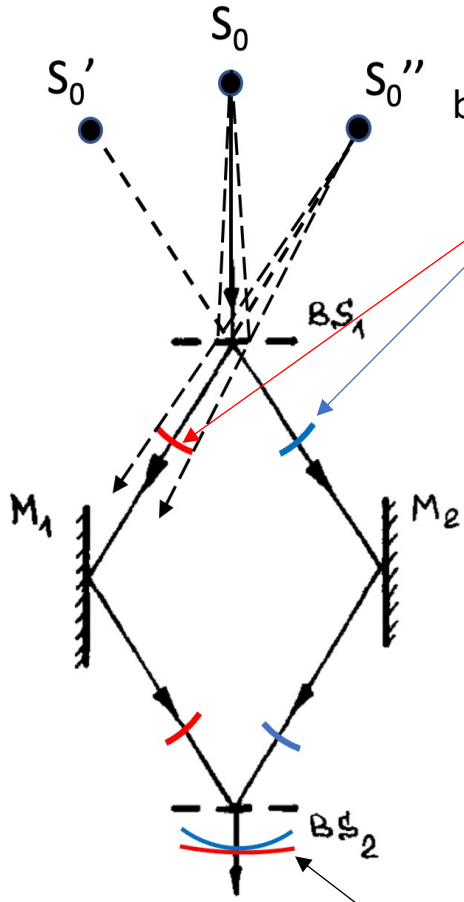
Idea: T. Ebisawa et al, NIM A 344, 597 (1994)



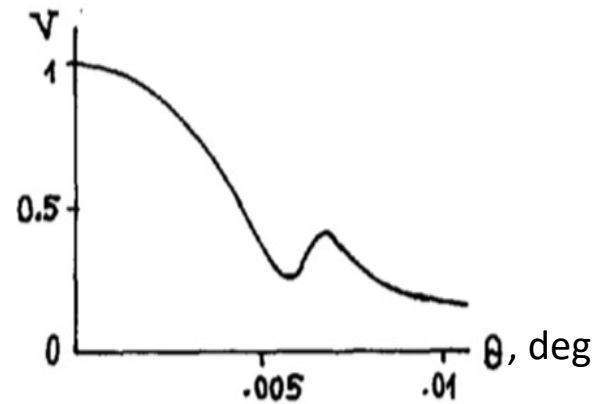
Angle between first and second pairs

See also presentation
of H. Shimizu
at 1st UCN/VCN workshop

3- grating interferometers



Spherical incident wavefronts diffracted by periodic structures are principally aberrated, but non-identical for $m=1$ and $m=-1$.



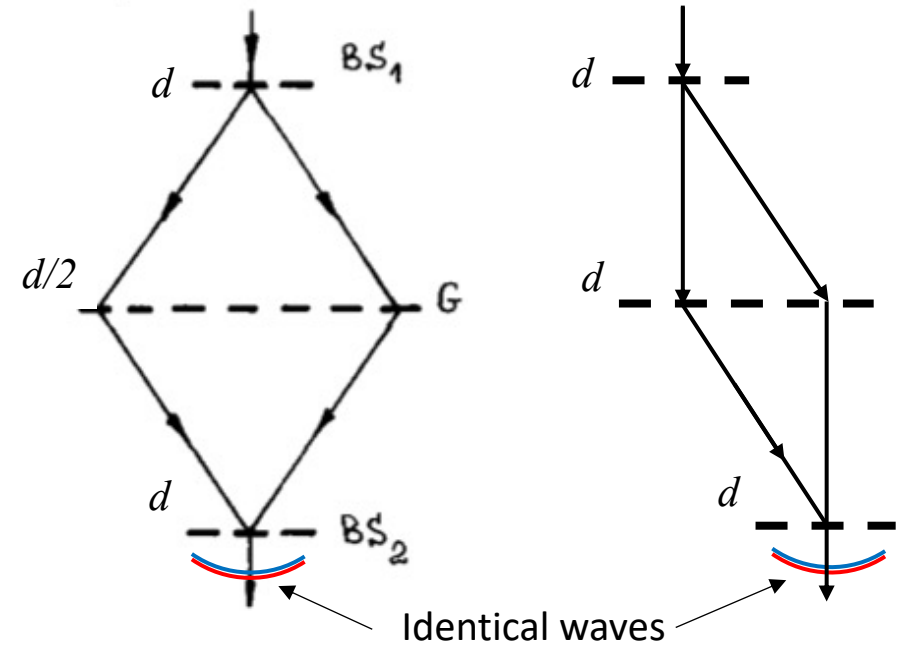
=> Strong requirements to incident beam divergence
=> Bad for neutrons in general; unfeasible for VCN

Interference of two non-identical waves:
=> non-constant period of the interference pattern
=> amplitude modulation over the beam cross-section
=> low visibility V

Unavoidable different aberrations in interfering beams:
=> add complimenting aberrations for equalization.

Deflection --> Diffraction: gratings instead of mirrors

3-grating interferometers

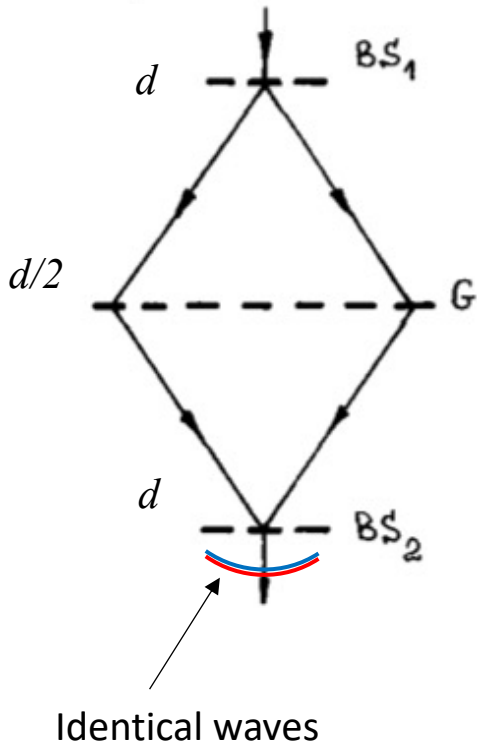


Aberration analysis shows that now interfering wavefronts are distorted identically and $V=1$:
=> no requirements to incident beam divergence

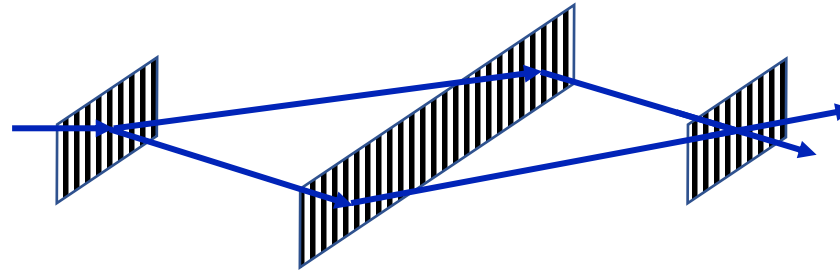
Diffraction grating interferometers

This is not the Talbot interferometer: Talbot effect is a near-field diffraction effect, where the self-imaging of periodic objects (gratings) **requires spatially coherent illumination**.

Here: the imaging of a grating by a second grating **regardless of the coherence of the source**.



- First shown by first-order diffraction theory (i.e. without accounting for aberrations): (*B.Chang, R.Alferness, E.Leith (Appl. Opt. 14 (1975) 1569)* .
- Aberration analysis (higher-orders diffraction theory): full compensation of aberrations => interfering waves are identical (*A.Ioffe, NIM A268 (1988) 169*).



Such interferometer works regardless of the source coherence, i.e. for non-monochromatic and non-collimated neutron beam!

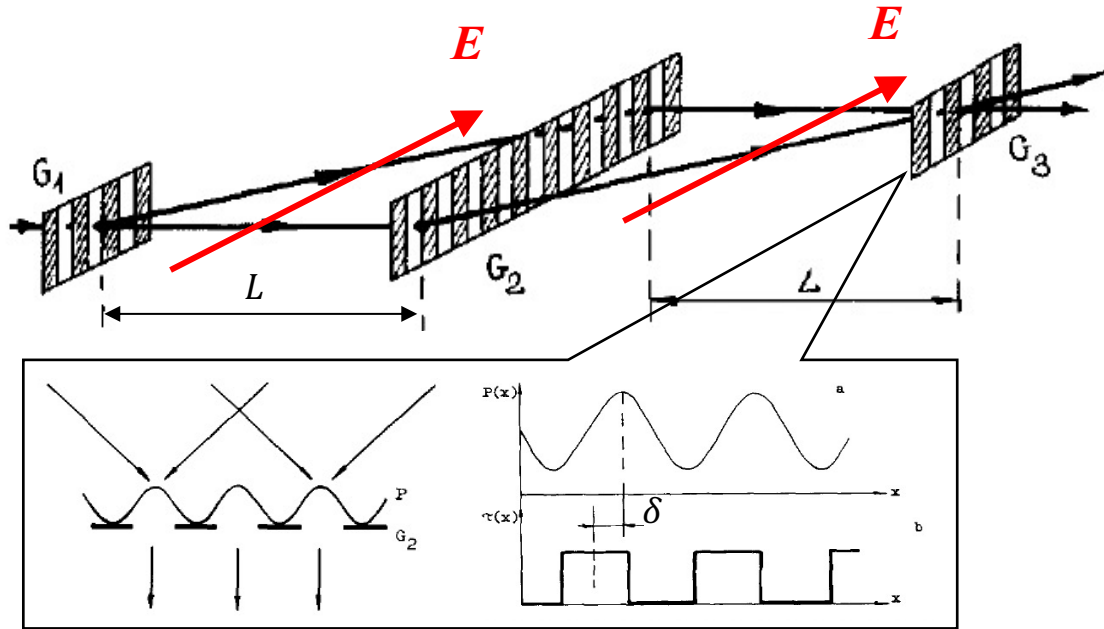
Transition to neutrons:
refraction index of vacuum in gravitational field $\neq 1$.
(*I.M Frank, A.I Frank, JETP Lett. 28 (1978) 515*)

$$n = \sqrt{1 - 2gz \left(\frac{m_n}{h}\right)^2 \lambda^2} \Rightarrow$$

As neutrons propagate on parabolic trajectories, vacuum has non-linear refraction index. This is not trivial, will be discussed later.

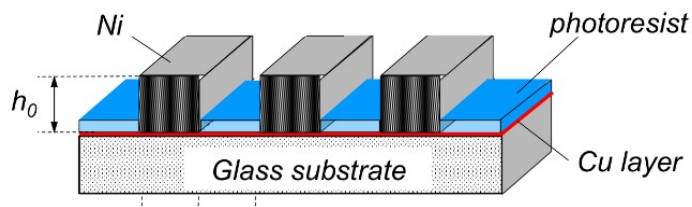
VCN diffraction grating interferometer for search of q_n

Electric field applied across interferometer beams



$$I(\Delta x) = I_0 V \left(1 + \cos \frac{2\pi}{d} \delta \right)$$

Phase diffraction gratings: surface relief



$d = 3.3 \mu\text{m}$
 $h_0 = 1.7 \mu\text{m}$: phase shift π for $\lambda = 20 \text{ \AA}$

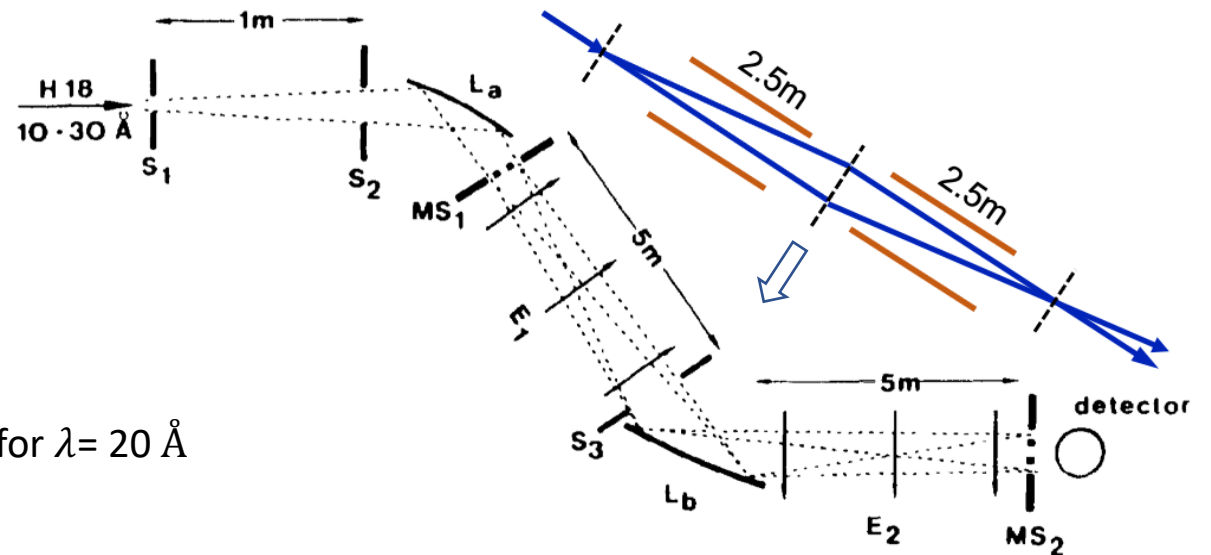
A. Ioffe, NIM A228 (1984) 141; NIM A268 (1988) 169.

$$\delta = \frac{1}{2} q E \left(\frac{L}{h} m \lambda \right)^2 \quad q = \frac{\sqrt{2} d}{\pi \sqrt{I_0}} \frac{1.5}{E L^2 \lambda^2} \left(\frac{h}{m} \right)^2$$

1987- proposal to ILL (accepted, but was not materialized):
 to use the same setup at H18 as for previous q_n experiment:

$I_0 = 200 \text{ n/s}$, $\lambda = (20 \pm 0.15) \text{ \AA}$, $E = 60 \text{ kV/cm}$, $L = 5 \text{ m}$

$q_n \geq 2 \cdot 10^{-22} e$ in 60 days - order of magnitude improvement



First realization of VCN diffraction grating interferometer

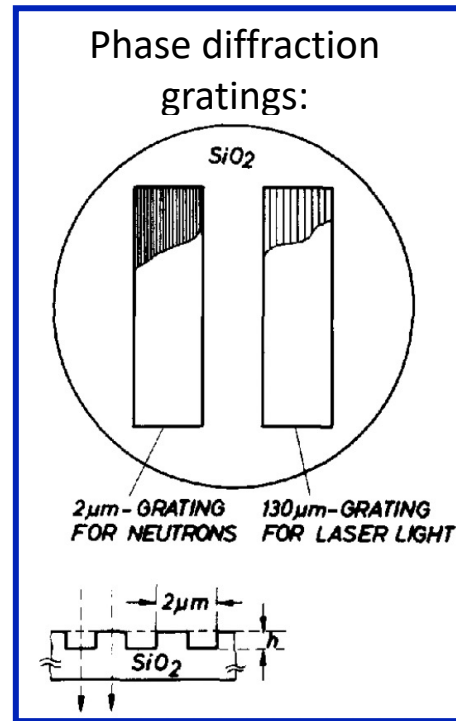
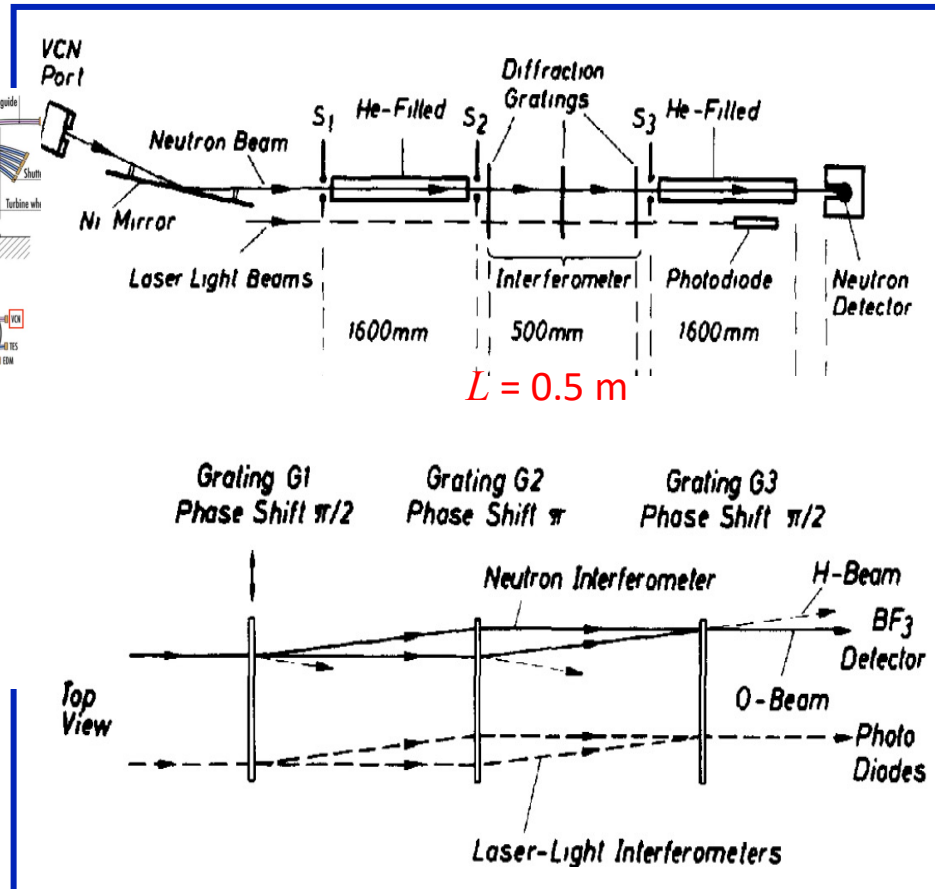
Volume 140, number 7,8

PHYSICS LETTERS A

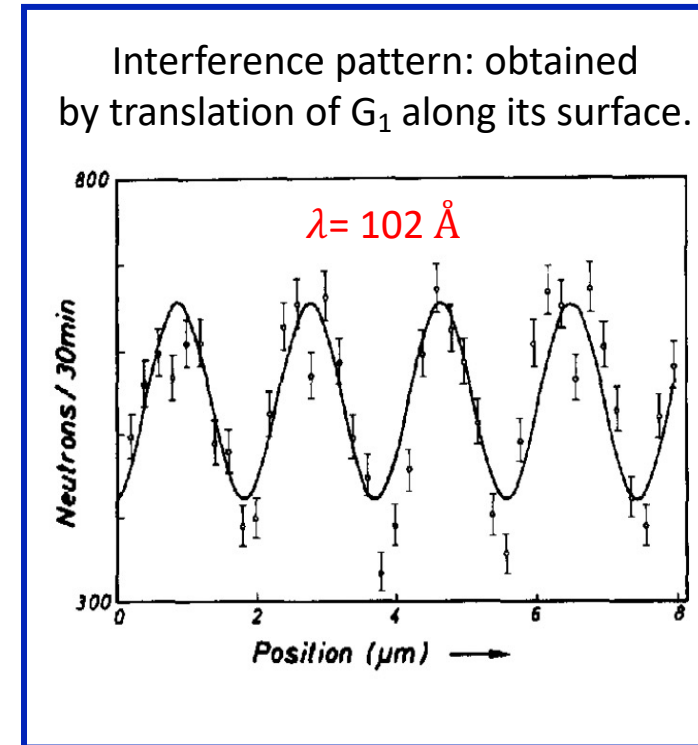
9 October 1989

A PHASE-GRATING INTERFEROMETER FOR VERY COLD NEUTRONS

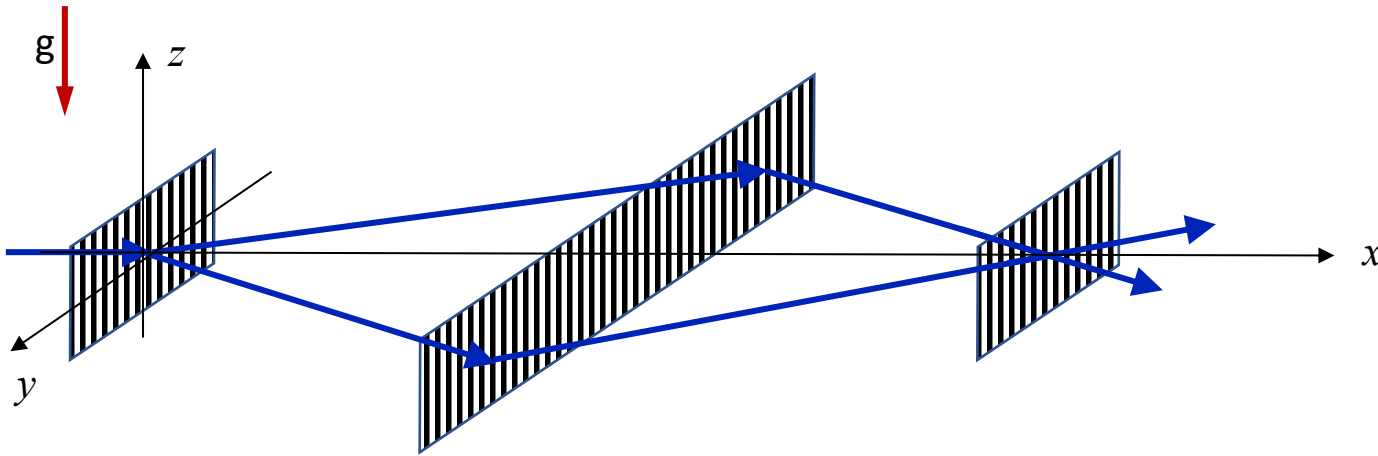
M.Gruber, K.Eder, A.Zeilinger, R.Gähler, W.Mampe



$d = 2 \mu\text{m}$



VCN diffraction grating interferometers in gravitational field

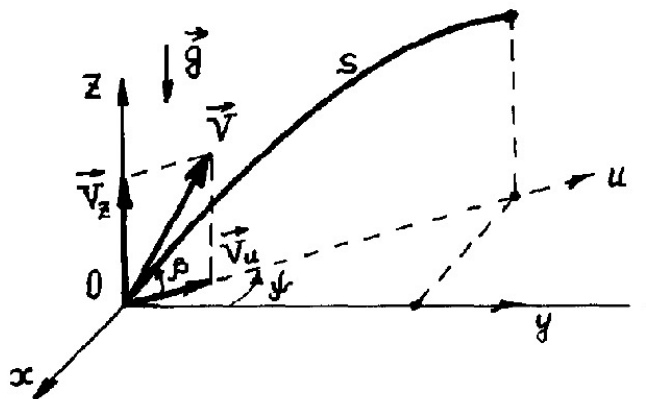


Refraction index of vacuum in gravitational field
(I.M Frank, A.I Frank, *JETP Lett.* **28** (1978) 515)

$$n = \sqrt{1 - 2gz \left(\frac{m_n}{h}\right)^2 \lambda^2}$$

Neutrons propagate on parabolic trajectories, i.e. in the media with non-linear refraction index.

Quasi-classical approximation: calculations of the phase of neutron wave, propagating over classical trajectory

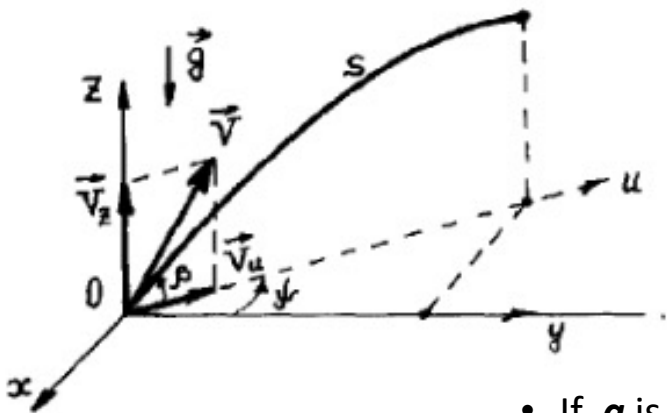


$$z = -\frac{gu^2}{2V_u^2} + \frac{V_z}{V_u} u, \quad \lambda(u) = \frac{h}{m_n} \left[V_u^2 + \left(V_z - \frac{gu}{V_u} \right)^2 \right]^{-1/2} \quad A.Ioffe, NIM A268 (1988) 169.$$

Gravitational phase shift:

$$\Phi = \int_0^{u_L} \frac{2\pi}{\lambda(u)} \sqrt{1 + \left(\frac{dz}{dx}\right)^2} du, \quad \varphi(u) = \frac{m_n}{\hbar} \left[uV_u + \frac{V_z^3}{3g} - \frac{V_u^3}{3g} \left(\tan \beta - \frac{gu}{V_u^2} \right)^3 \right],$$

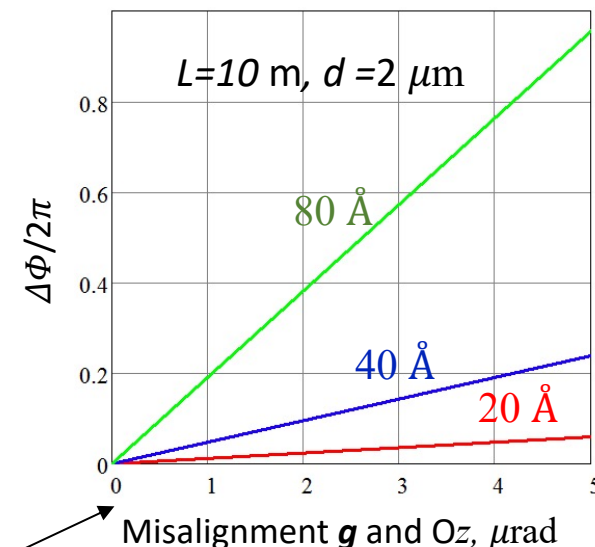
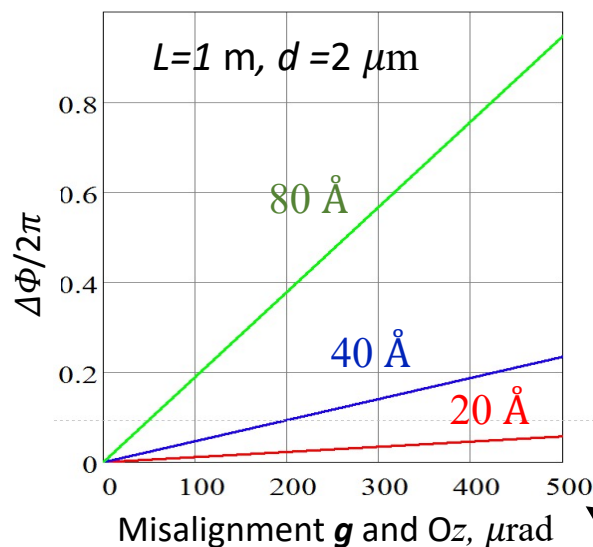
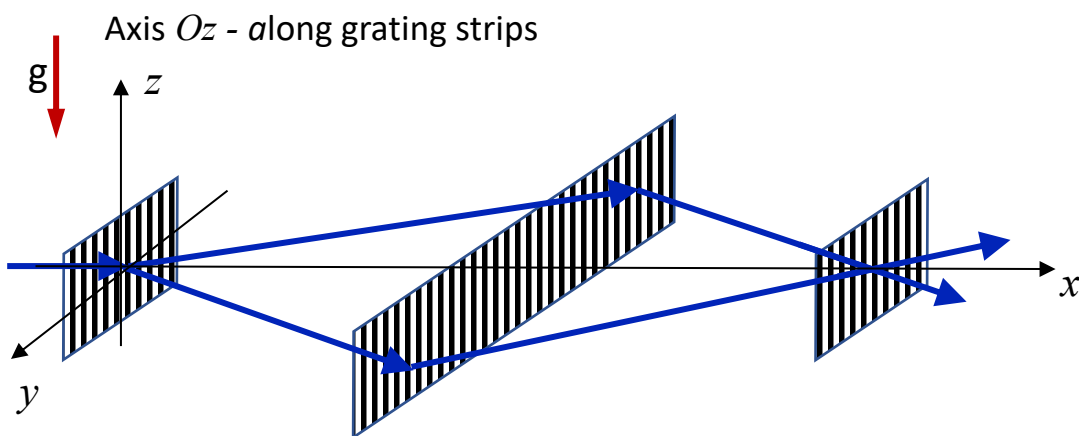
VCN grating interferometers in gravitational field



$$\Phi(u) = \frac{m_n}{\hbar} \left[uV_u + \frac{V_z^3}{3g} - \frac{V_u^3}{3g} \left(\tan \beta - \frac{gu}{V_u^2} \right)^3 \right],$$

Calculating the velocity components immediately after diffraction, it is possible to calculate the phase shift of neutron wave during its following propagation.

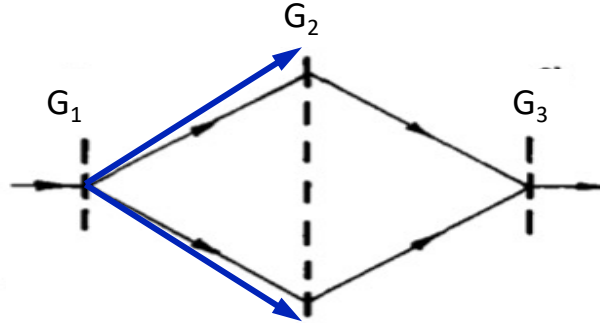
- If \mathbf{g} is strictly parallel to Oz (grating strips), gravitational potentials for both sub-beams are equal (symmetry).
- Violation of this symmetry leads to neutron trajectories rising to different heights => phase difference.



Note different scales!

Symmetric 4-grating interferometer

Sagnac effect

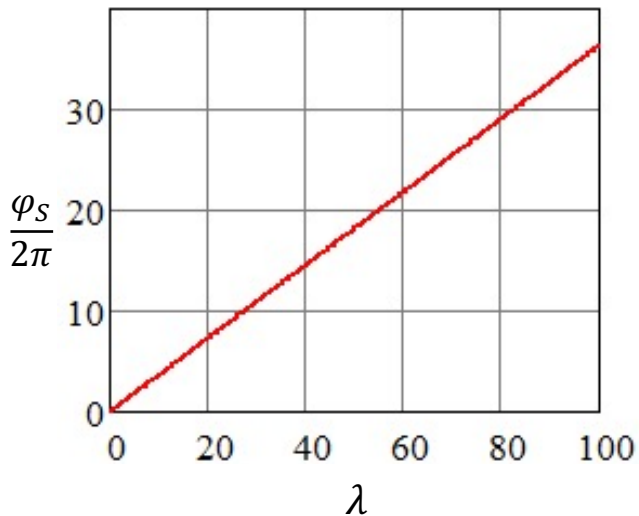


- (+) for VCN: aberration-free, V=100% for full incoherent illumination
- (-) for VCN: requires μrad alignment relative \mathbf{g}
- (-) parasitic Sagnac effect

$$\varphi_S = \frac{2m_n}{\hbar} (\boldsymbol{\omega} \cdot \mathbf{A}) = \frac{2m_n}{\hbar} \omega_0 A \sin \theta_1,$$

$\omega_0 = 7.29 \cdot 10^{-5} \text{ s}^{-1}$ angular velocity (Earth's rotation)

$\theta_1 \approx 56^\circ$ - latitude angle (Lund)



$A = \frac{\lambda L}{d 2}$ area enclosed by interfering beams

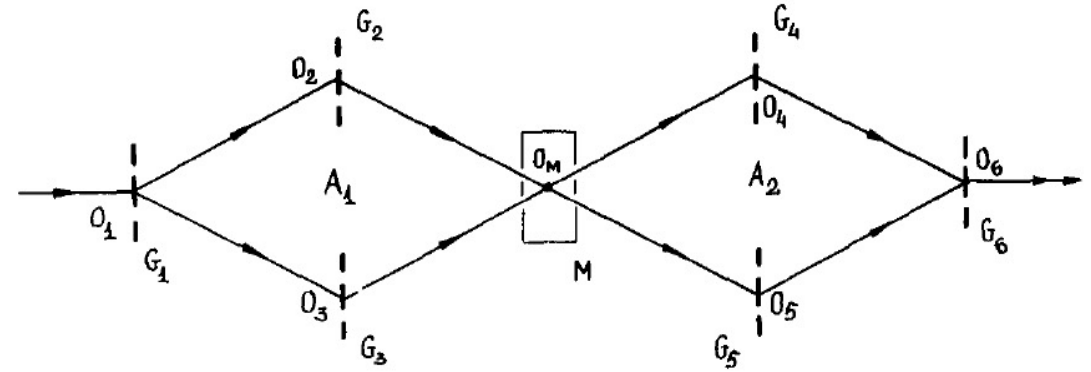
$\Delta\lambda \rightarrow \Delta A$ Scatter in λ

$$\Delta\varphi_S \sim \Delta A = \frac{\Delta\lambda}{\lambda} A = 0.1A$$

For $\Delta\varphi_S > \pi/2$ interference fringes are washed out.

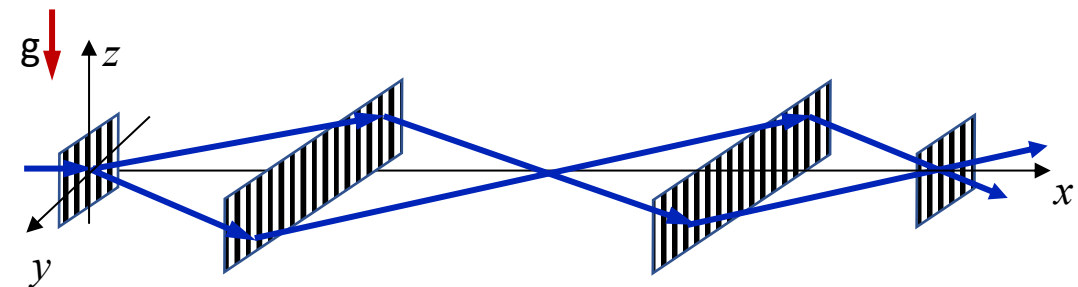
Symmetric 4-grating interferometer

A.Ioffe, NIM A268 (1988) 169.



$A_1 = A_2$ (symmetry) => Complete compensation of Sagnac phase shift

Also complete compensation of gravitational phase difference

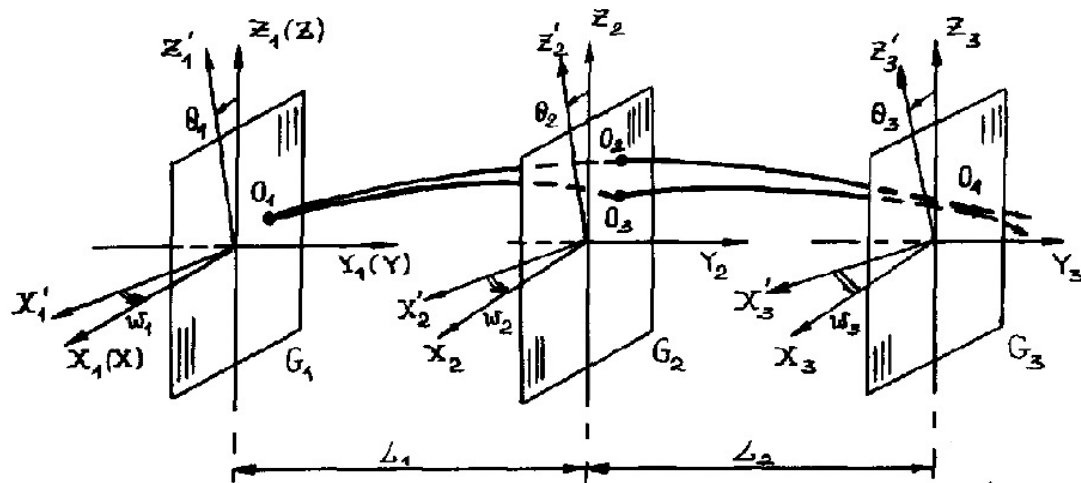


Not for free: one more grating - additional intensity losses

VCN grating interferometer: adjustment

Ray tracing (not Monte Carlo): regular grid defined by Shannon-Kotelnikov theorem, rather than random grid.

Non-parallelism of grating planes



For each ray (neutron) and both interferometer arms:

1. Vector V of initial neutron velocity is defined in the laboratory frame XYZ (OZ parallel g)
2. Components of V are transformed to coordinate frame of grating G_1 by the Eulerian rotational matrix
3. Velocity vector components after diffraction are determined from diffraction grating equation.
4. Phase shift $\Phi(u)$ for propagation over path to G_2 is calculated.

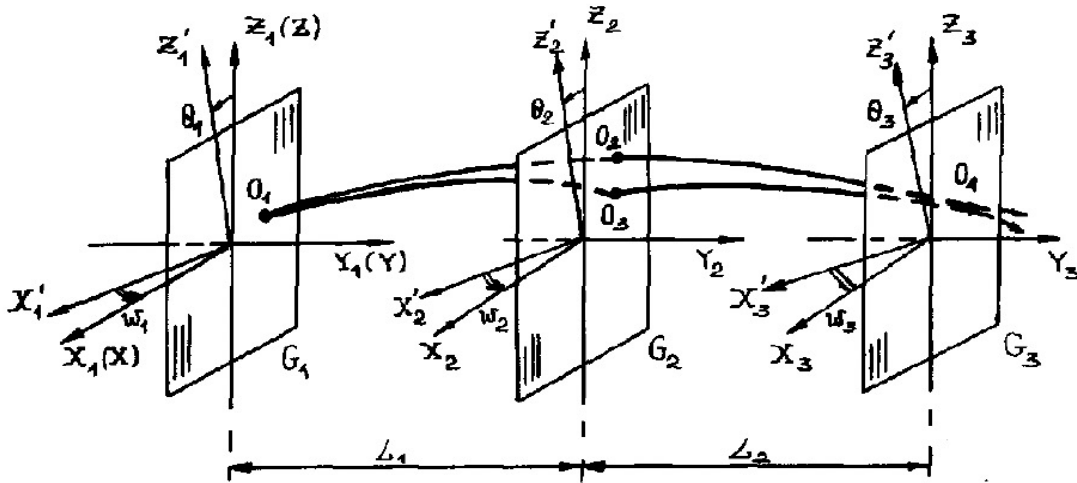
$$\Phi(u) = \frac{m_n}{\hbar} \left[uV_u + \frac{V_z^3}{3g} - \frac{V_u^3}{3g} \left(\tan \beta - \frac{gu}{V_u^2} \right)^3 \right],$$

VCN grating interferometer: adjustment

Ray tracing (not MC): regular grid defined by Shannon-Kotelnikov theorem, rather than random grid.

Non-parallelism of grating planes leads to:

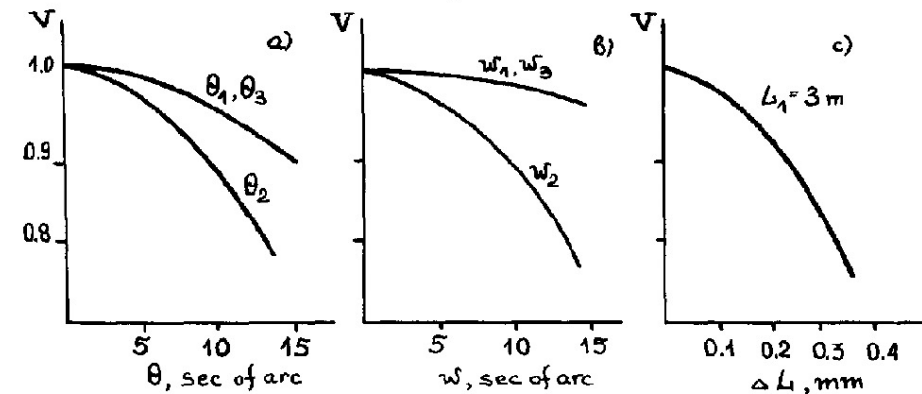
- Path difference and spatial separation between interfering rays after diffraction on grating G_3
- Appearance of interference fringes in the output beam cross-section => reduced visibility.
- Visibility defines requirements to adjustment accuracy



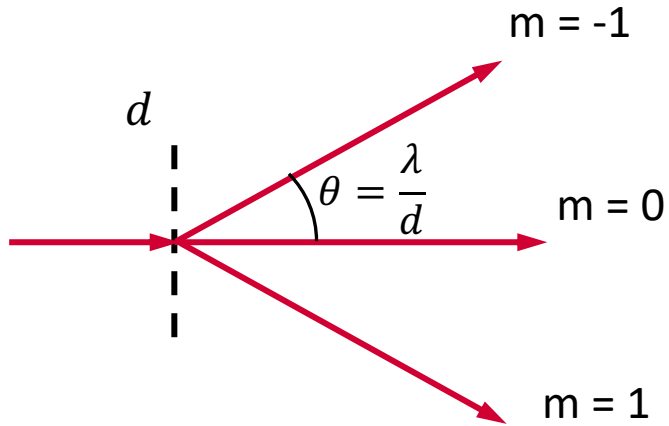
For $L=6\text{m}$, $\lambda=100\text{ \AA}$:
angular accuracy $< 10''$, $\Delta L < 0.15\text{ mm}$

Not a complicate technical problem.

Dependences of Visibility on misalignment



Diffraction gratings for VCNs

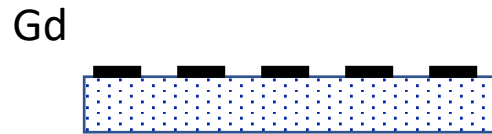


Requirements: small period and high diffraction efficiency

- Photolithographic gratings (stamping in photoresist)
- Holographic photolithographic gratings (interference lithography)
- Holographic nanodiamond-polymer composite gratings (next talk by J.Klepp)

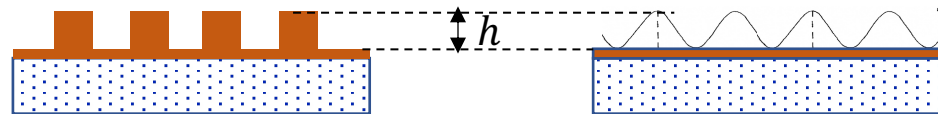
Phase gratings

Amplitude gratings



Phase shift: $\varphi = \frac{2\pi\lambda}{\rho} h$

ρ - scattering length density
 $h = 1.74 \mu\text{m}$ for $\lambda = 20 \text{ \AA}$.



Diffraction efficiency:

$$\eta_m = \frac{1}{\pi^2 m^2}$$

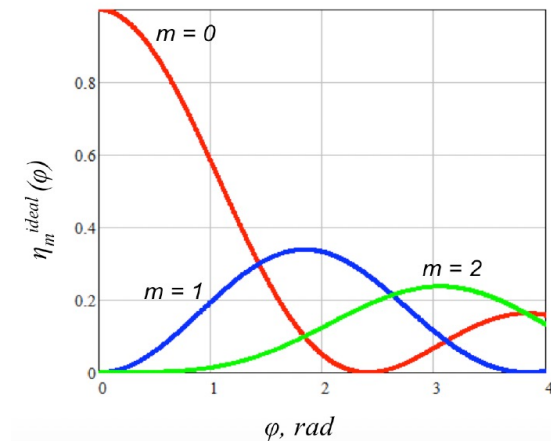
$$\eta_1^{\text{max}} = 10.1\%$$

$$\eta_m = \frac{4}{\pi^2 m^2} \sin^2(\varphi)$$

$$\eta_1^{\text{max}} = 40.4\%$$

$$\eta_m = [J_m(\varphi)]^2$$

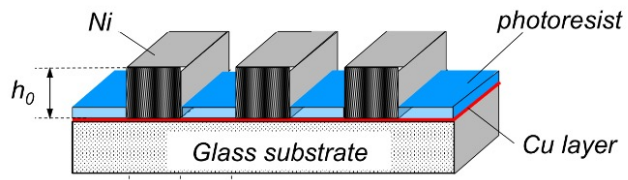
$$\eta_1^{\text{max}} = 33.8\%$$



Diffraction gratings for VCNs

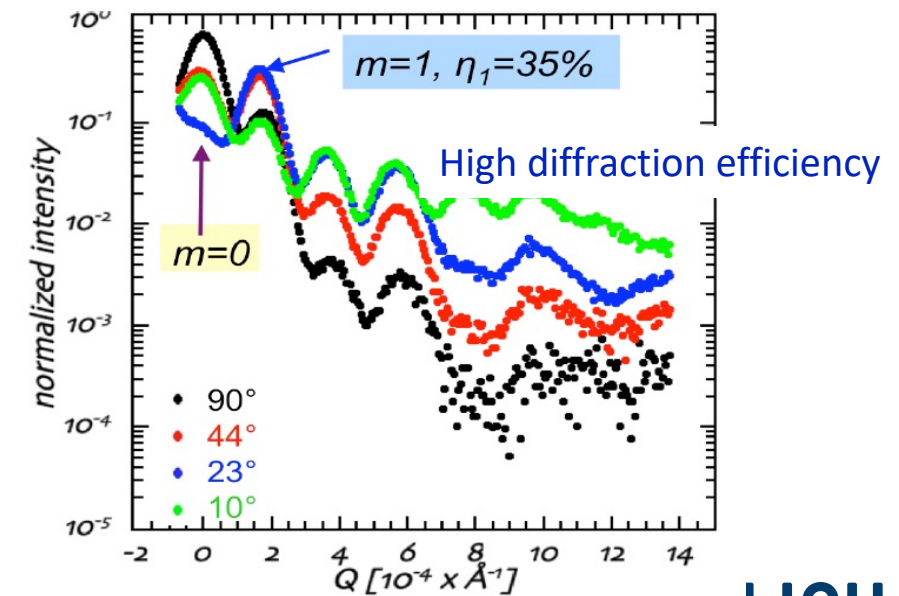
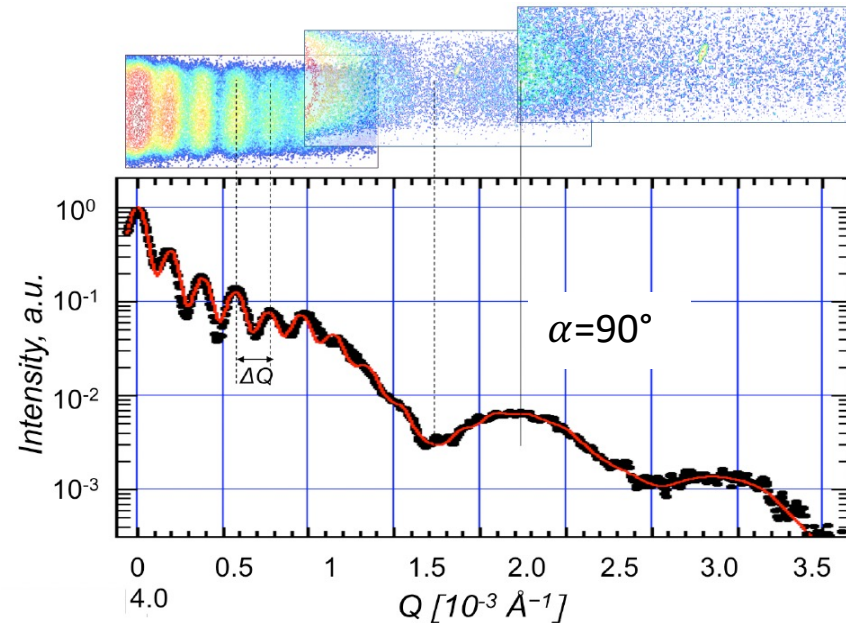
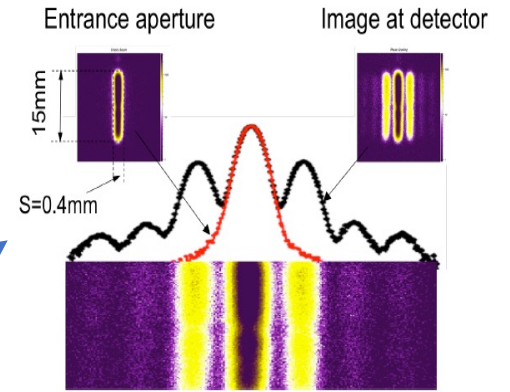
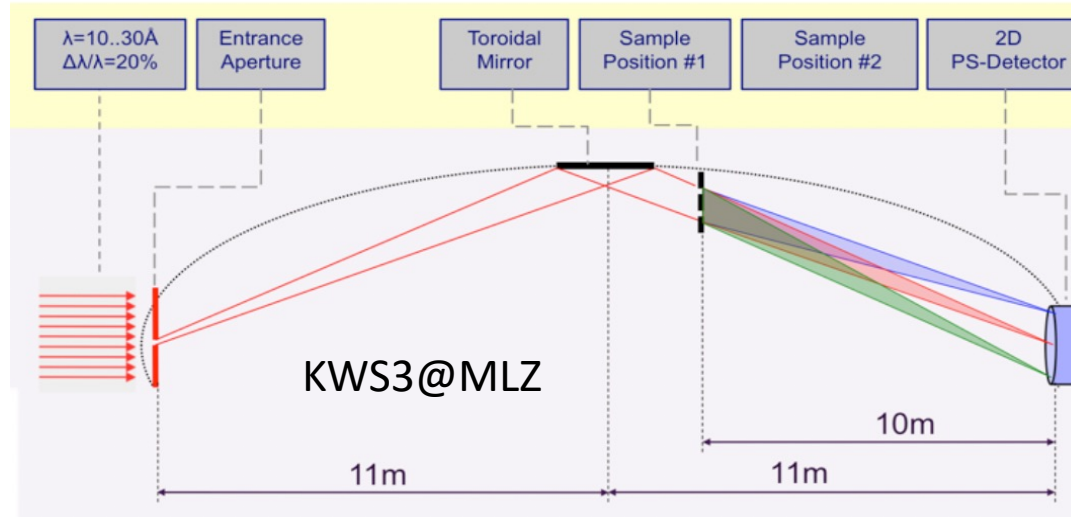
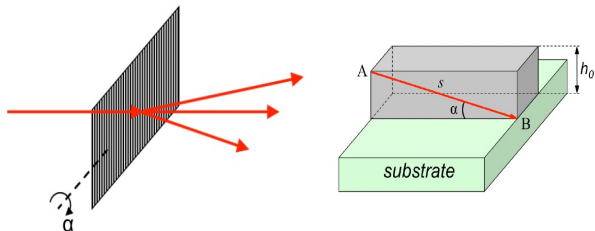
A. Ioffe, V. Pipich, JPS Conf. Proc.22, 011014 (2018)

Electrochemical deposition through PR mask

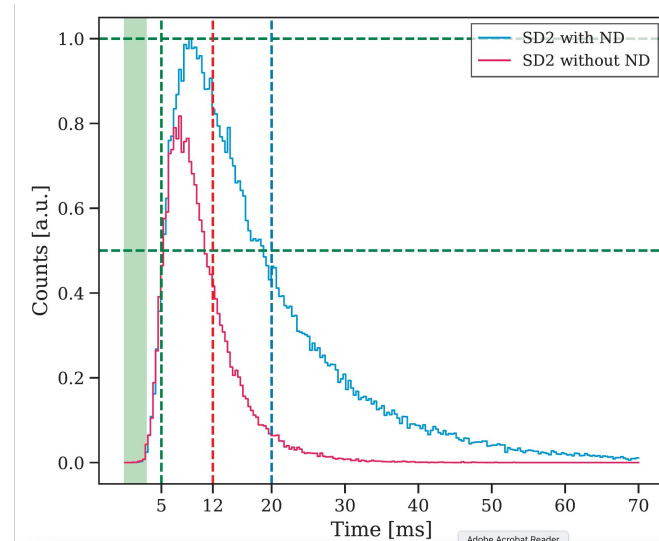
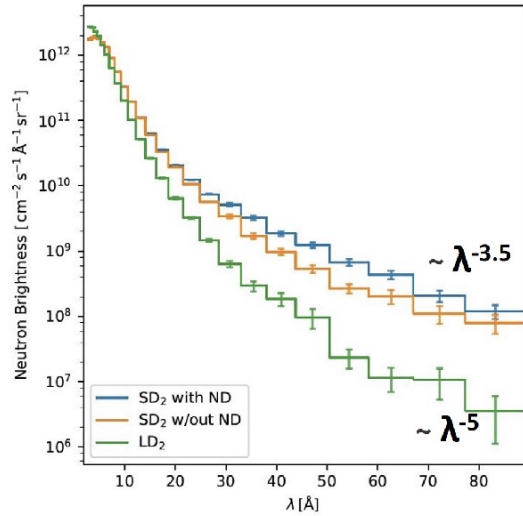


$d = 3.3 \mu\text{m}$, $h_0 = 1.7 \mu\text{m}$
 phase shift $\varphi = \pi$ for $\lambda = 20 \text{ \AA}$
 (ILL experiment)

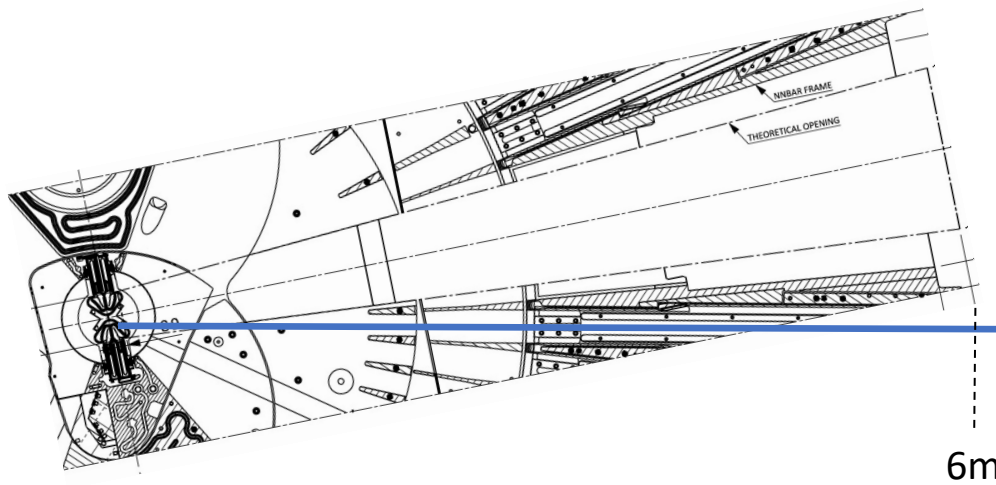
KWS3: $\lambda = 12.6 \text{ \AA} \Rightarrow$ tilt
 $\varphi / \sin(\alpha) = \pi$



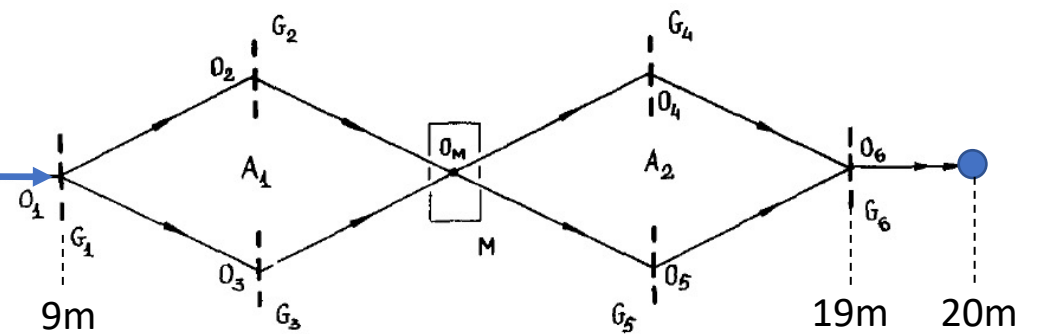
VCN diffraction grating interferometer at ESS



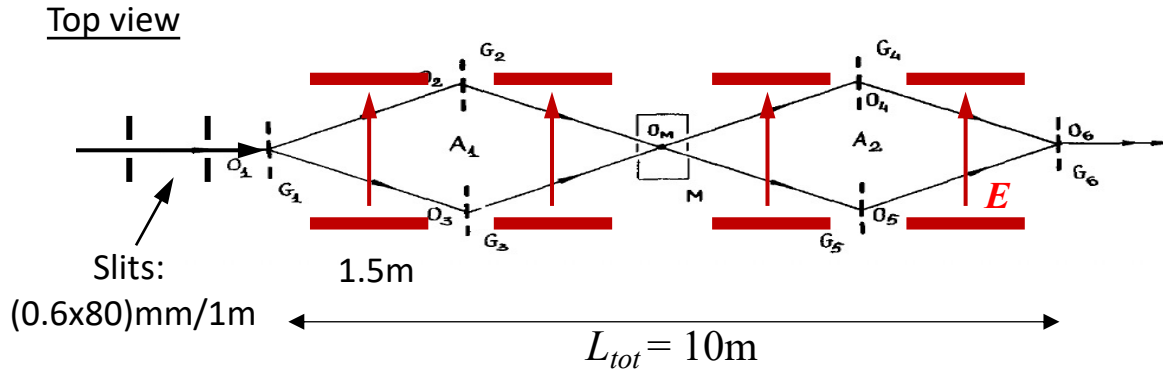
Data from L. Zanini



Not in scale



VCN diffraction grating interferometer at ESS: search for q_n



$$d = 2 \mu\text{m}$$

$$V=50\%$$

$$E = 60 \text{ kV/cm}$$

$$L_E = 6m$$

Beam parameters: the same as at the ILL setup

Beam cross-section: $S_{beam} = 0.48 \text{ cm}^2$

Solid angles: $\omega_x = 0.0006$

$$\omega_y = 0.08 \text{ (} L=10m, \text{ last slit is } G_4 \text{)}$$

$$\Delta\lambda = 3956 \cdot T_{rep} / L_{source-det} = 14 \text{ \AA}$$

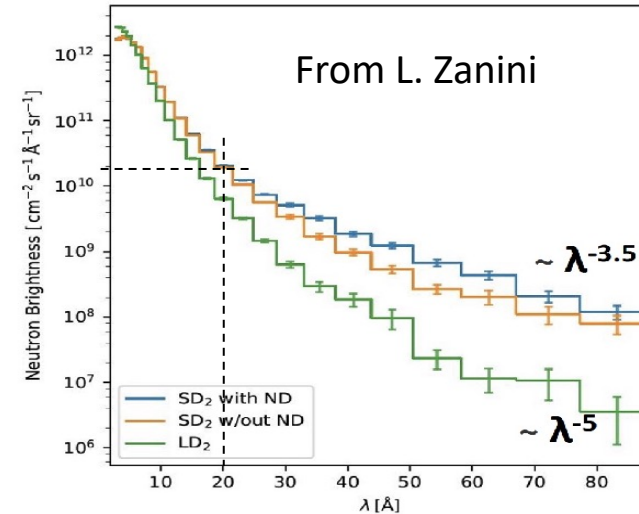
Diffraction efficiency: $\eta^4 = 0.008$ ($\eta = 30\%$)

Transmission of substrates ($\lambda = 20 \text{ \AA}$):

$$\text{Si } 4 \times 0.07 \text{ cm: } T_{\text{Si}} = 0.93$$

$$\text{SiO}_2 \text{ } 4 \times 0.3 \text{ cm: } T_{\text{SiO}_2} = 0.63$$

$$I_{rec}(\lambda) = B(\lambda) S_{beam} \omega_x \omega_y \eta^4 \Delta\lambda T_{\text{SiO}_2}$$

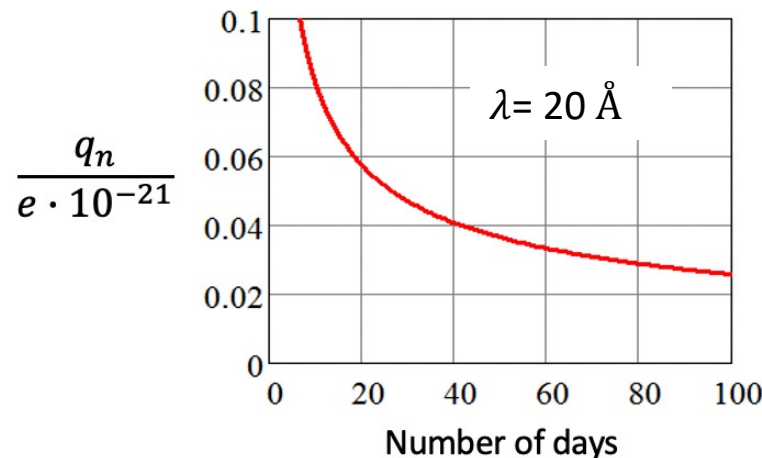


$$B(20 \text{ \AA}) = 2 \times 10^{10}$$

Expected counting rate:

$$I_{rec}(20 \text{ \AA}) \approx 4.9 \cdot 10^3 \text{ n/s}$$

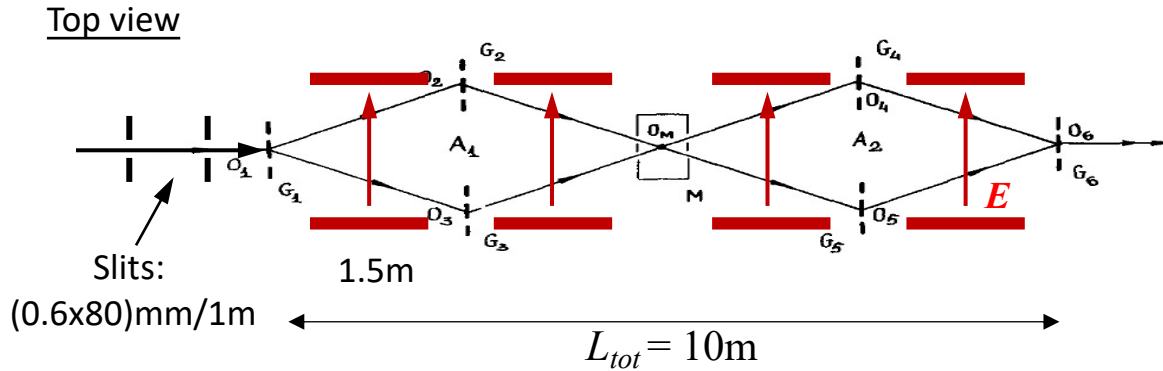
$$q(\lambda) = \frac{\sqrt{2} d}{\pi \sqrt{I_{rec}(\lambda) 8.64 \cdot 10^4 N_{days}}} \frac{1.65}{V E L^2 \lambda^2} \left(\frac{h}{m}\right)^2$$



$$q_n \geq 3 \cdot 10^{-23} e \text{ in 80 days (CI 90\%)}$$

2 orders of magnitude better,
than the present day limit

VCN diffraction grating interferometer at ESS: search for q_n



Transition to higher λ : does it make sense?

$$q_n \sim \frac{d}{\sqrt{I_0 E L^2 \lambda^2}}$$

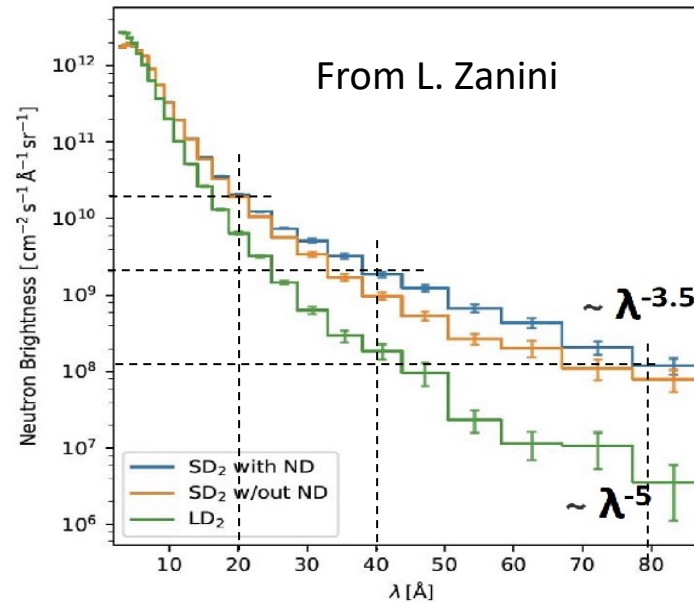
LD₂: $q_n \sim \frac{1}{\lambda^{-2.5} \lambda^2} \sim \lambda^{0.5}$ Getting worse

SD₂: $q_n \sim \frac{1}{\lambda^{-1.75} \lambda^2} \sim \lambda^{-0.25}$ Getting better

SD₂ is a game changer.

Transition from 20 Å to 80 Å gives factor of 2 improvement:

$q_n \geq 1.5 \cdot 10^{-23} e$ in 80 days (CI 90%)

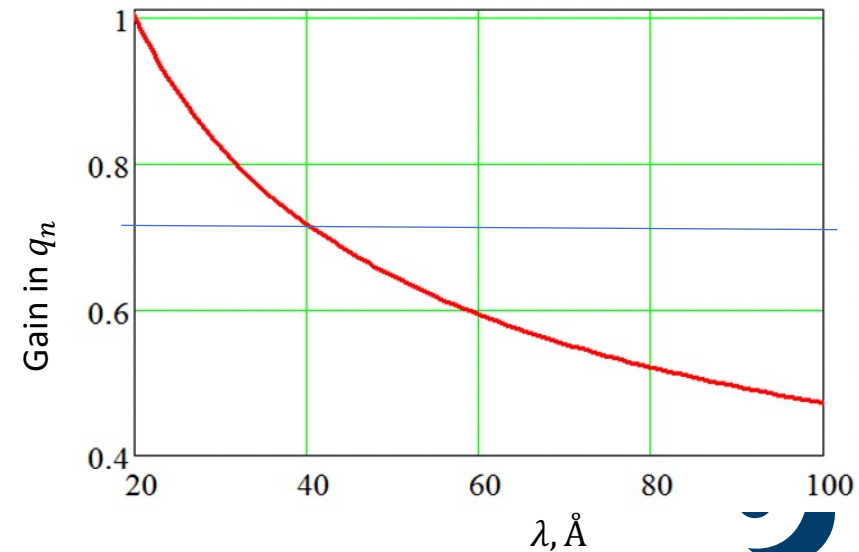


Expected counting rate:

$I_{rec}(20 \text{ Å}) \approx 4.9 \cdot 10^3 \text{ n/s}$

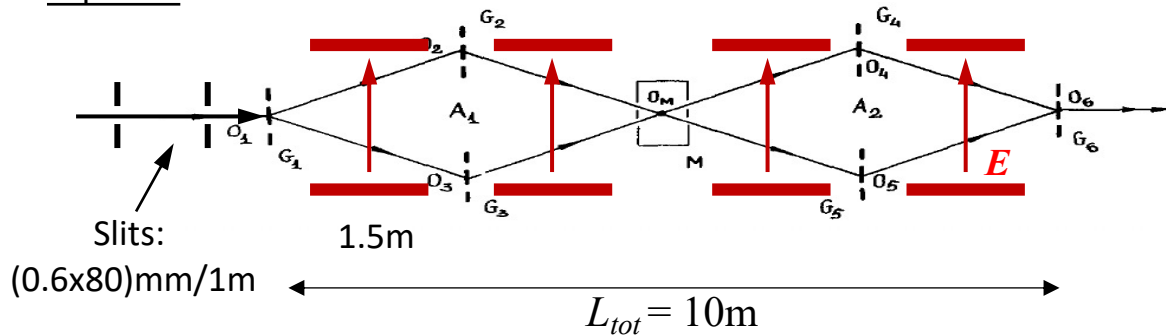
$I_{rec}(40 \text{ Å}) \approx 4.9 \cdot 10^2 \text{ n/s}$

$I_{rec}(80 \text{ Å}) \approx 4 \cdot 10^1 \text{ n/s}$



Potential for further improvements in search for q_n

Top view



$$q_n \sim \frac{d}{\sqrt{I_0} E L^2 \lambda^2}$$

Practically, only “free” parameter is grating period d .

Reducing the period d of diffraction gratings to sub- μm :

\Rightarrow direct gain as $q_n \sim d$

\Rightarrow increase of diffraction angle $\theta = \frac{\lambda}{d}$,

therefore gain in incident beam intensity $\sim d^2$:

gain in solid angle $\omega_x \sim d$ (still $\ll \lambda/d$)

gain in beam cross-section $\sim d$

Overall gain in $q_n \sim d^{-2}$

$V=50\%$, $E=60$ kV/cm, $L_E=6$ m

Beam parameters:

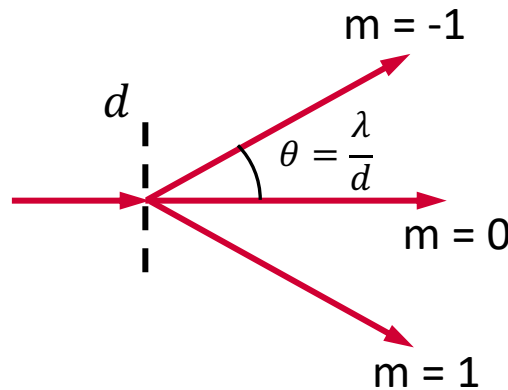
Beam cross-section: $S_{\text{beam}} = 0.48$ cm²

Solid angles:

$\omega_x = 0.0006 \ll \lambda/d = 0.005$

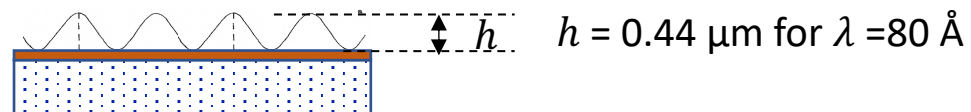
$\omega_y = 0.08$ ($L=10$ m, last slit is G_4)

Band: $\Delta\lambda=14$ Å



$d: 2 \mu\text{m} \rightarrow 0.5 \mu\text{m}$ results in additional improvement by an order of magnitude: $q_n \geq 10^{-24} e$

\Rightarrow Holographic (interference) gratings



Conclusion

- Interferometry of cold, especially Very Cold Neutrons (VCN) requires diffraction gratings for effective coherent splitting of neutron waves.
- Diffraction gratings introduce distortions (aberrations) in propagating waves, that however can be compensated in 3-grating neutron interferometer. Such interferometer works regardless of the source coherence, i.e. for **non-monochromatic and non-collimated neutron beams**.
- The Earth gravitational field causes additional aberrations of neutron waves. Moreover, the Earth rotation results in an additional phase shift (Sagnac effect). Each of these makes large VCN interferometers unfeasible.
- Symmetric 4-grating interferometer allows for the full compensation of both above mentioned effects.
- Such interferometer can be used for the neutron charge quest. Being installed at a new high-brilliance VCN source at ESS it will allow to improve the present day experimental limit on neutron charge by 2 orders of magnitude, down to $3 \cdot 10^{-23} e$.
- The use of holographic (interference) gratings with sub- μm period should allow for additional gain of about 10.

Thank you for attention!