# Effect of an Iron Yoke of the Field Homogeneity in a Superconducting Double-Helix Bent Dipole

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**Abstract:** Charged particle accelerators require large dipole fields with stringent homogeneity requirements needed to bend particle beams without defocussing. Commonly superconducting saddle coil magnets are used with an iron core to enhance the bore field. The iron uneven magnetization brings undesired multipole fields that need to be compensated for by pre-conditioning the beam with additional magnets. Since the beam is bent by a dipole field, it would be desirable to have bend magnets following the trajectory of the beam. Bending saddle coils dramatically distort the bore field generating undesired multipole fields costly to compensate for. A bent iron core also significantly contributes to the multipole content of the bore field, its effect strongly depending on its shape and saturation state. AML's Double-Helix (DH) winding technology allows for the design of magnetic field with arbitrary multipole content through precise positioning of conductors, thus making compensation for any field inhomogeneity practical. The paper presents the analysis of the effect of the iron core on the field homogeneity of a 3.5T bent dipole built at the Advanced Magnet Lab and the optimized winding leading to a pure dipole field.

**Keywords:** Bent dipole, iron saturation, multipole field compensation.

## 1. Introduction

Applications for charged particle accelerators are becoming more numerous every day from medical applications such as proton therapy for cancer treatment to industrial material processing. Bending charged particle beams requires a magnetic dipole field with very good field homogeneity of typically $10^{-4}$; indeed, any higher order magnetic multipole fields have undesired effects on the beam such as defocussing. The angle at which the particle beam is bent depends on the beam stiffness (energy) and the applied magnetic field. In an attempt to make particle accelerators more compact, beams have sometimes to be bent on 180° making the dipole magnets design very challenging. Ideally, bent dipole magnets following the trajectory of the beam should be used. However, bending a dipole magnet creates unwanted multipole field that have to be compensated for. Additionally, iron yokes are used to contain the magnetic flux and enhance the dipole field; their non-uniform magnetization and possible saturation also create undesired multipole fields. While designing and building a bent dipole based on conventional saddle coils is very challenging and highly impractical, the intrinsic design flexibility of Double-Helix™ (DH) magnet technology allows for a perfect control of the multipole content of the field and thus any unwanted multipole order of the field can be effectively and easily cancelled as part of the magnet design. The work presented in this paper deals with the identification of the multipole fields stemming from the non-uniform magnetization of iron yokes in a straight and bent dipole magnet using COMSOL MultiPhysics.

### 3. Double-Helix Magnets

Double-Helix™ magnet technology allows for the generation of magnetic multipoles with unmatched field homogeneity [1], [2], [3]. Intrinsically, because the conductor distribution forms an almost perfectly sinusoidal current distribution, field homogeneity better than $10^{-4}$ can be achieved. DH magnets can achieve high field homogeneity thanks to a manufacturing process that stabilizes the conductors in precisely machined grooves. Combined function magnets can be developed within a single winding such as a superimposition of several multipole orders and/or twisting or bending. This unique capability is achieved without affecting the field homogeneity. The double-helix coil configuration uses concentric pairs of oppositely-modulated helical windings to generate transverse magnetic fields. Figure 1 shows a 2-layer magnet generating a transverse dipole field.

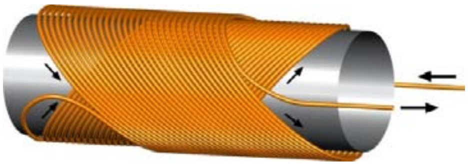

**Figure 1**. Example of a 2-layer winding used to form a DH dipole magnet

The DH solenoid-like windings are imbedded in concentric cylinders of high-strength material. Together with an overbanding of high-modulus, high-strength fibers, forces can be contained easily. The current follows a path governed by the following equation:

$$X(\theta) = \frac{h}{2\pi}\theta + \sum_n \frac{a}{\tan(\alpha_n)} \sin(n\theta + \varphi_n)$$
$$Y(\theta) = a\cos(\theta) \qquad (1)$$
$$Z(\theta) = a\sin(\theta)$$

The different parameters of equation (1) are defined in figure 2.

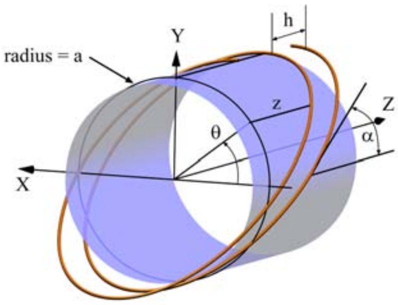


**Figure 2**. Geometric properties of DH winding

Any mathematical transformation can be applied to equations (1) leading to modifications of the current paths and therefore of the generated magnetic field. DH magnets can be optimized through the adjustment of the values of the turn advance, the modulation amplitude, the number of turns and the “tilt” angle, as defined in figure 2, for each layer.
Figure 3 shows a bent dipole magnet based on Double-Helix™ technology in which all the undesired multipole orders stemming from the deformation are eliminated. The superconducting magnets was built, as shown in figure 4, and successfully tested.

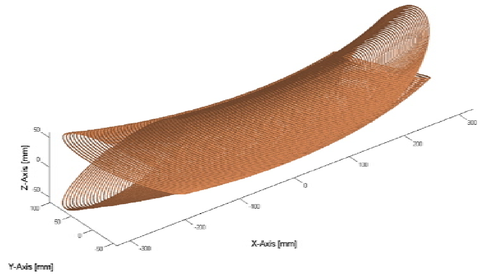

**Figure 3**. Bent DH dipole magnet generating a homogeneous field

**Figure 4**. Bent DH dipole magnet being manufactured at AML

## 4. Dipole Magnet Specifications

A DH superconducting dipole bent magnet was designed according to specifications for a medical application requiring a large dipole field in the 3.8 T range. The magnet specifications are summarized in table 1.

**Table 1:** Superconducting Bent Dipole Magnet Specifications

| | |
|---|---|
| Aperture | 250 mm |
| Nb. Of layers | 10 |
| Conductor (cable) OD | 1.5 mm |
| Current | 1000 A |
| Dipole field | 3.8 T |
| Axis radius | 2 m |

## 5. Straight Magnet

We will start by modeling a straight dipole magnet and assess the effect of the iron yoke on the field uniformity. Since DH magnets are able to generate very pure field distributions, they are accurately modeled by ideal sinusoidal current density distributions. The end effects are neglected and a 2D model is adequate to perform the analysis of interest. Figure 5 shows the model as implemented in the COMSOL graphical interface in 2D. The magnet current density is assumed to be sinusoidally distributed: $J_x=J_{max}*\cos(\mathrm{atan2}(y,x))$, $J_y=J_{max}*\sin(\mathrm{atan2}(y,x))$.

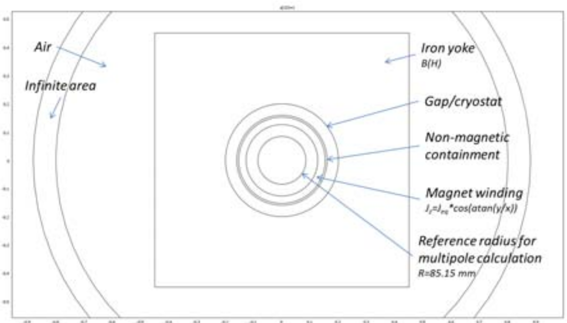


**Figure 5**. 2D model of a perfect dipole magnet

### 5.1 Perfect Dipole Magnet without Iron Yoke

The designed magnet generates a dipole bore field of 2.62 T that will be enhanced by a non-saturated iron yoke to about 3.8 T. As shown in figure 6 and 7, the field in the bore of the magnet is perfectly uniform.

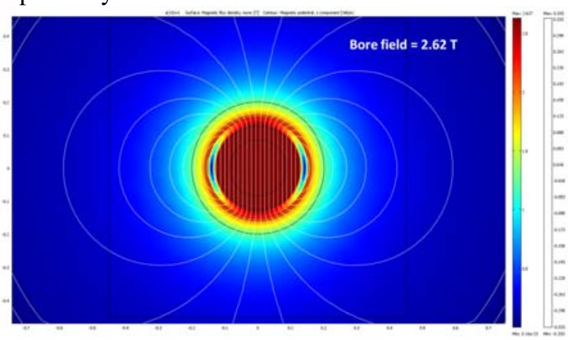


**Figure 6**. Flux density distribution in a perfect dipole magnet in air.

Figure 7 shows the magnitude of the magnetic flux density along the x direction through the axis of the magnet. The flux density distribution is symmetrical and the magnitude of *B* in the bore is constant. Any higher order multipole field would distort the bore field that would present some variations. The multipole analysis is performed using a Fourier decomposition of the field along a reference radius of 85.15 mm in the bore of the magnet. The analysis is done for the first 15 multipole orders.

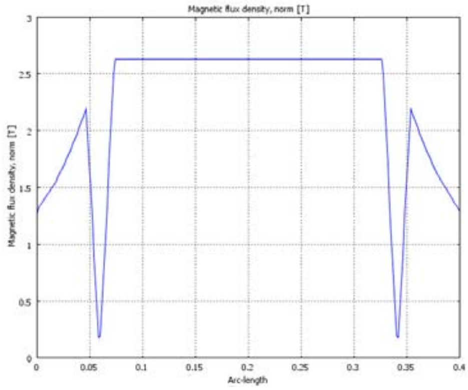


**Figure 7**. Magnetic flux density in the bore of a perfect dipole magnet

### 5.2 Effect of Iron Yoke on Field Uniformity

The dimensions of the iron yoke are determined so that the peak field remains around 2 T for the nominal current of 1000 A. Staying right below saturation of the magnetic material allows for a good magnetic shielding and for an acceptable linearity of the bore field vs. the magnet excitation current as shown in figure 9. The dimensions of the yoke are shown in figure 8. The bore field is enhanced to 3.85 T with the iron yoke.

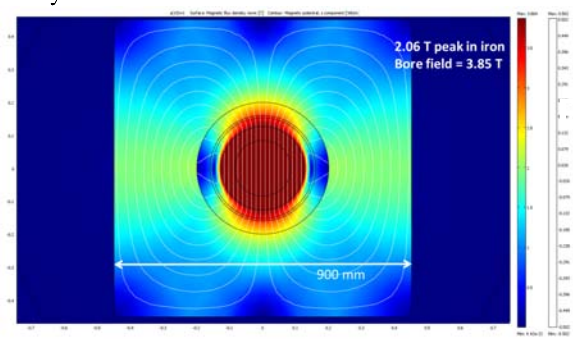


**Figure 8**. Perfect dipole magnet with iron yoke

As shown in figure 8, the magnetization of the iron yoke is not uniform; this non-uniformity creates higher multipole fields in the magnet bore. The difference of magnetization between the vertical and horizontal parts of the yoke depends on the excitation current of the magnet. Figure 10 shows the relative variation of magnitude of the created multipole fields.

The iron yoke magnetization creates both a sextupole (b3) field up to 0.08% and a decapole (b6) field up to 0.035% of the dipole field at nominal current. While those values seem small, their effect on a charged particle beam can be significant and usually, field homogeneity of $10^{-4}$ are required.

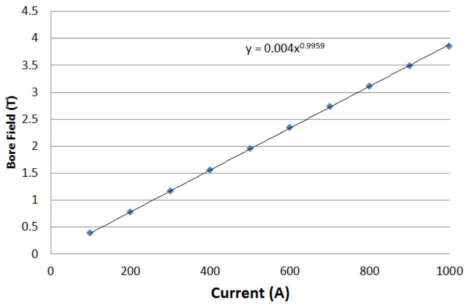


**Figure 9**. Bore field vs. excitation current

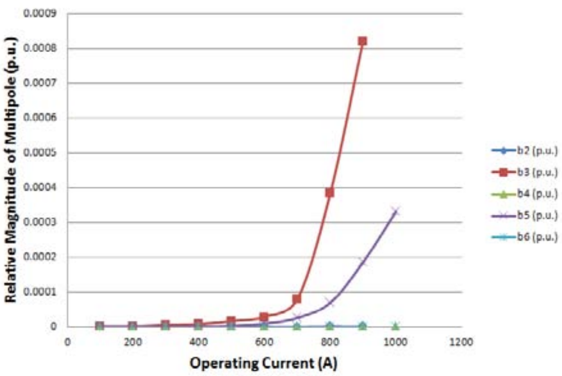


**Figure 10**. Variation of the multipole orders vs. excitation currents

## 5.3 Effect of Iron Saturation on Field Uniformity

Figure 11 shows the flux distribution in a dipole magnet with a saturated yoke; the shielding of the field is partial leading to an important stray field. Also, as the saturation level of the yoke increased, the bore field decreases as shown in figure 12.

The multipole content varies with the saturation level of the yoke. Figure 12 shows the relative evolution of the multipole fields as a function of the peak flux density if the iron yoke.

The iron saturation leads to a sextupole (b3) of up to 1.1% and a decapole (b5) field of up to 0.15 % of the dipole field. As the field increases in the iron, it becomes more “uniformly” magnetized thus lowering the multipole fields.

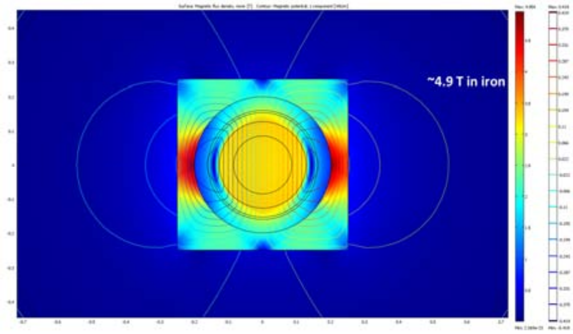


**Figure 11**. Flux distribution on a highly saturated iron yoke

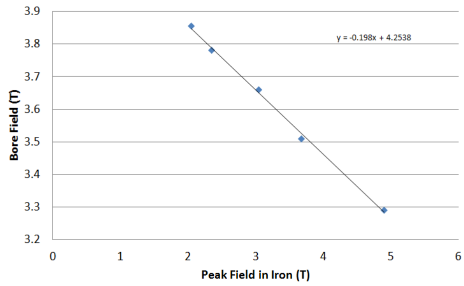


**Figure 11**. Bore field vs. peak field in the iron yoke

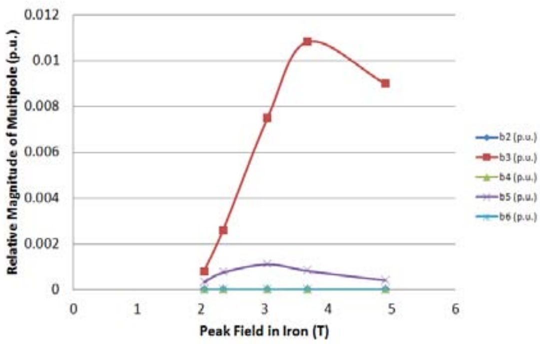


**Figure 12**. Relative evolution of the multipole components vs. peak flux density in the iron yoke

# 6. Bent Magnet

The straight dipole magnet presented in the previous sections of the paper is bent so that its axis follows an arc with a 2 meter radius.

## 6.1 Modeling 3D or 2D

As for the straight magnet, a 2D model can be used to assess the influence of the iron yoke. The end effects are neglected. As shown in figure 13, the 3D bent magnet was simulated using a 2D geometry with axial symmetry in COMSOL

MultiPhysics. The implemented geometry is shown in figure 14.

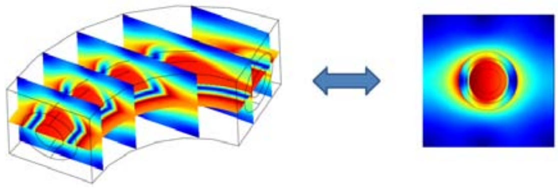

**Figure 13**. Model used is 2D axi-symmetrical

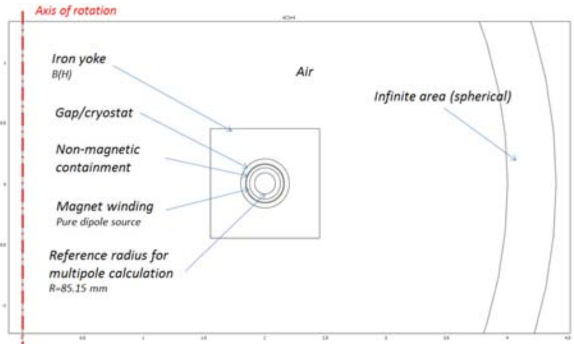


**Figure 14**. Geometry implemented in COMSOL

### 6.1 Perfect Bent Dipole Magnet without Iron Yoke

Bending the dipole magnet leads to a modification of the current density distribution with more current at the inside of the curvature and less at the outer part of the curvature [4], [5]. This has an influence on the bore field and creates a strong quadrupole (b2) field of ~1% and a sextupole (b3) component of ~0.03% of the dipole field. Current is adjusted in the model to compensate for the multipole content ($<10^{-6}$) allowing for the effects of the iron to be isolated.

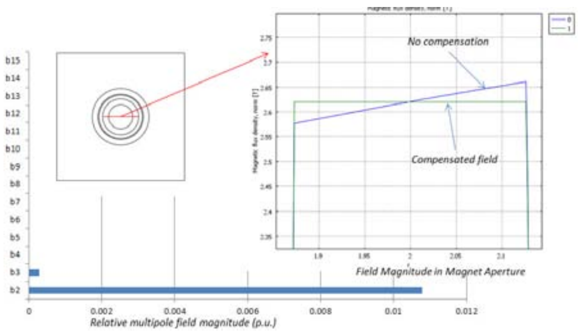


**Figure 15**. Compensation of the multipole fields stemming from the bending of the magnet

### 6.2 Influence of Bent Iron Yoke on Field Uniformity

The symmetry of the magnetization of the iron yoke observed for the straight magnet is no longer present in the bent iron yoke. The side presenting the smaller radius has a larger magnetization than the other side as shown in figure 16. The lack of symmetry has consequences on the bore field that is no longer uniform as shown in figure 17.

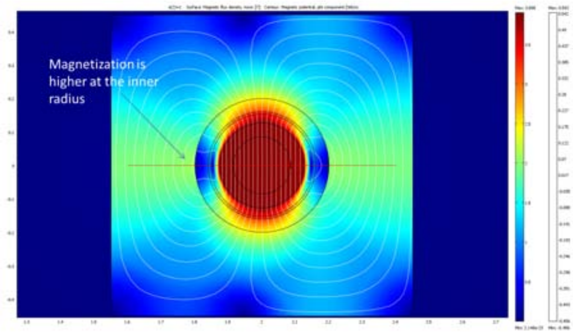


**Figure 16**. Magnetization of a non-saturated bent iron yoke

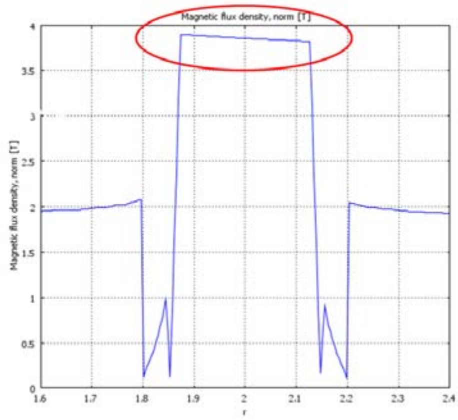


**Figure 17**. Bore field distortion due to the non-uniform magnetization of the iron core

The iron yoke is designed so that the peak value of the flux density does not exceed 2 T allowing for the bore field to evolve linearly with the excitation current as shown in figure 18.
The multipole fields created by the bent iron yoke are presented in figure 19. Without saturation, the bent iron yoke creates a strong quadrupole field (b2) up to 9% at low field and a strong sextupole (b3) field up to 0.22% of the dipole field. The quadrupole and decapole fields become significant after 800 A (0.02%). The magnitude of the multipole fields created by the bent iron yoke is about 10 times larger than that

of the straight magnet. Moreover, operation at variable beam energy levels requires operation at different field magnitudes which would require actively controlled compensation coils to cancel the multipole fields that depend strongly on the excitation current.

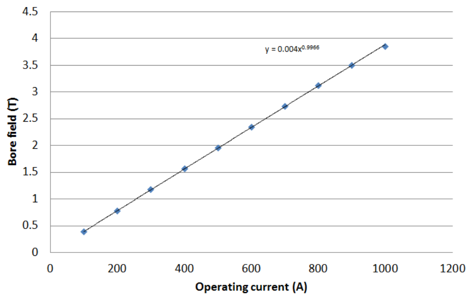


**Figure 18**. Bore field vs. excitation current showing that there is no noticeable saturation effect

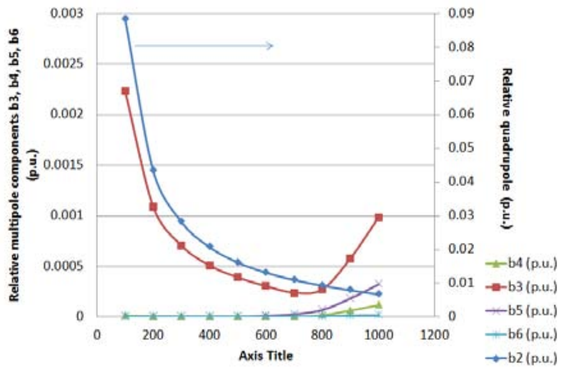


**Figure 19**. Magnitude of the multipole content of the bent magnet vs. excitation current

## 6.2 Influence of Bent Iron Yoke Saturation on Field Uniformity

If the iron yoke is undersized to save weight, the saturation will modify the difference of flux density magnitudes leading to a different multipole content. An example is shown in figure 20.

As for the straight magnet, the bore flux density decreases with the thickness of the iron yoke because of the saturation level of the iron. The effect is almost linear as shown in figure 21.

Figure 22 shows the influence of the yoke saturation on the multipole content. The iron saturation leads to a sextupole (b3) of up to 1.1%, a quadrupole field (b2) of up to 0.7 %, a decapole (b5) field up to 0.1% and a small octopole (b4) appears at high saturation. As the field increases in the iron, it becomes more "uniformly" magnetized thus lowering the multipole fields.

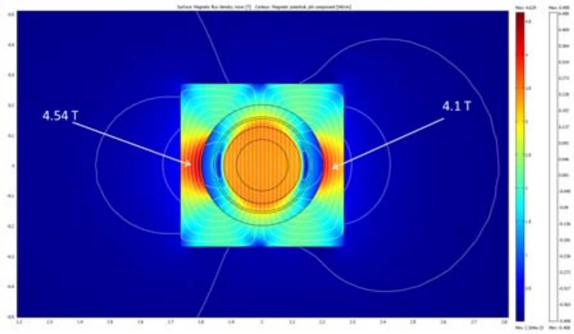


**Figure 20**. Saturated bent iron yoke.

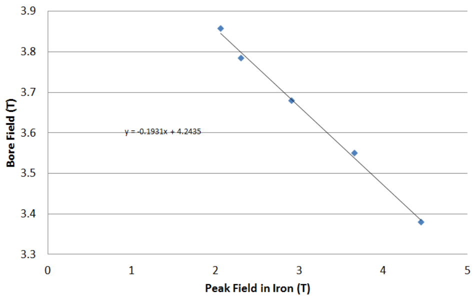


**Figure 21**. Bore field vs. peak field in the iron yoke

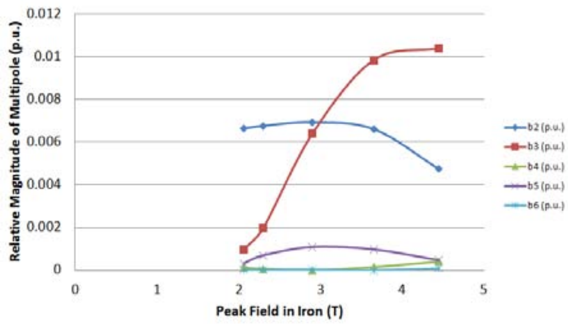


**Figure 22**. Magnitude of the multipole content of the bent magnet vs. peak field in the iron yoke

## 7. Conclusions

Through the use of COMSOL MultiPhysics, the influence of the iron yoke in straight and bent dipoles has been better quantified allowing for better magnets to be built.

The multipole fields created by the bent iron yoke are opposite to the ones created by the bent dipole magnet leading to some natural cancellation of some of the undesired effects. The work presented allows for the visualization of the effects of the iron yoke.

Because of its shape, the iron yoke of a bent dipole has a much stronger effect on field uniformity than a straight one. The multipole fields created would have a significant effect on the beam and need to be compensated. The magnitude of the multipole fields depends strongly on the operating current which makes active compensation necessary. Because of its intrinsic design flexibility, Double-Helix winding technology enables the development of bent dipoles with active compensation making possible more compact particle accelerators.

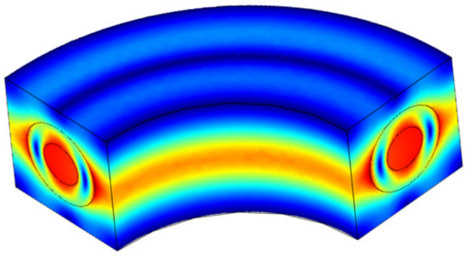

**Figure 23**. Flux density distribution in a bent dipole magnet with iron yoke

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