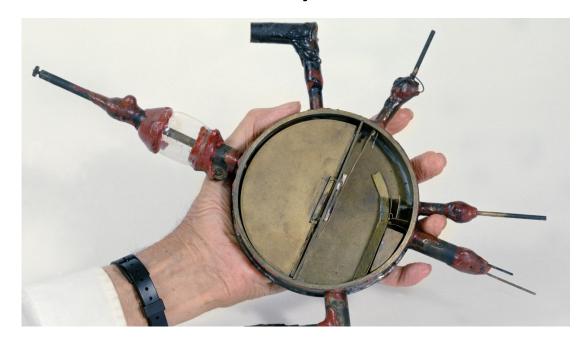
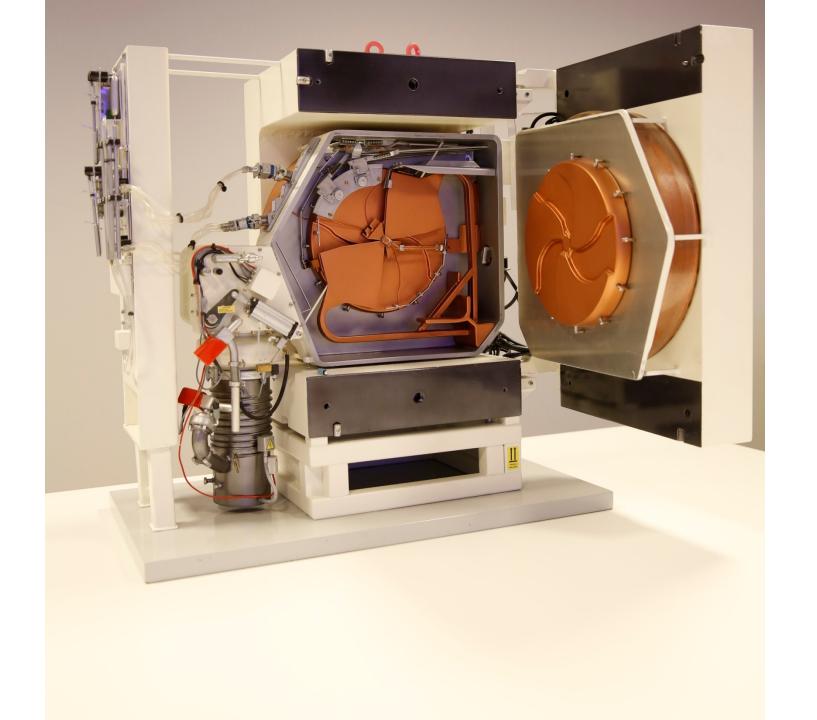
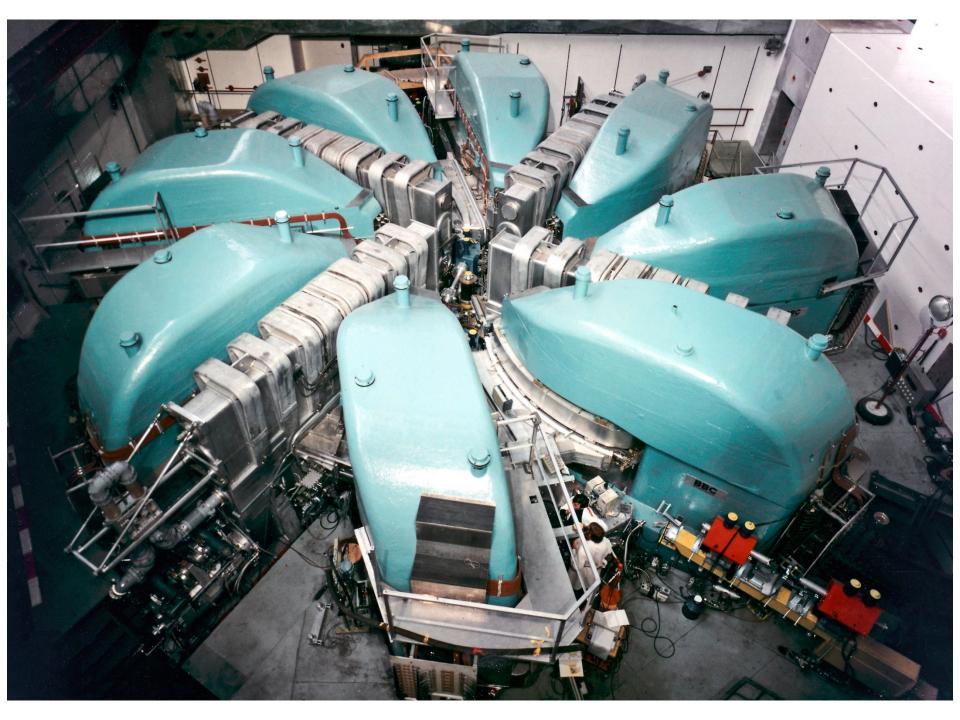
Cyclotrons

- Classical cyclotron
- Synchrocyclotron
- Isochronous cyclotron

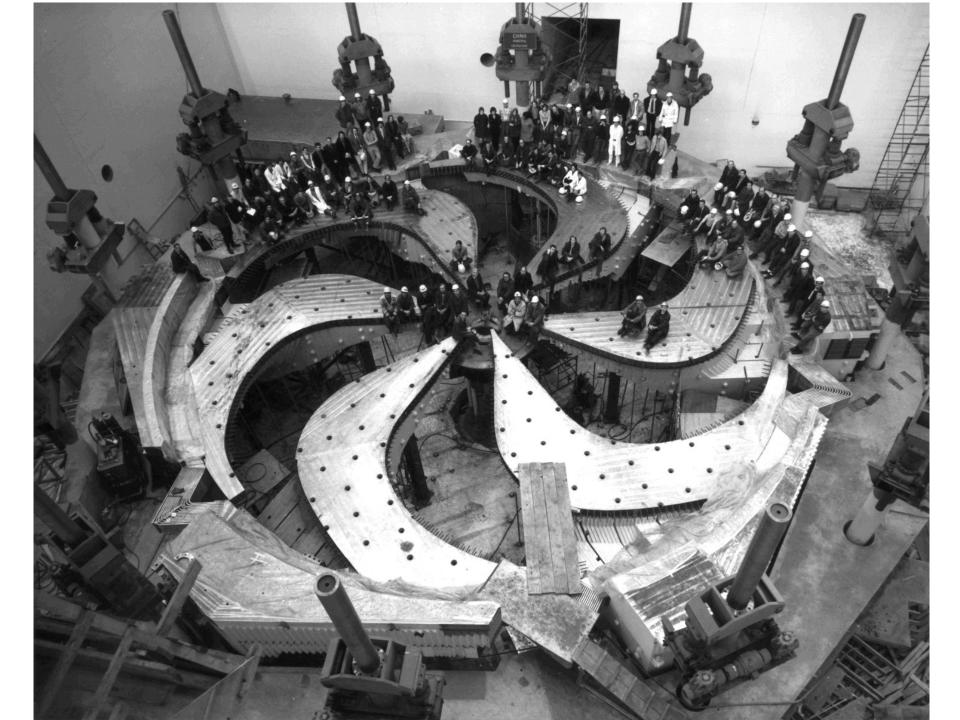


Ernest O. Lawrence, 1932 Nobel Prize, 1939









Classical Cyclotron

(Brianstant)

-a charged particle (q,m) in a magnetic field (B)

V上3

centripetal force = magnetic force

$$\Rightarrow \frac{\sqrt{2}}{r} = \frac{48}{m}$$

w=we= cyclotron frequency

tre= wc = accelerating frequency = wer

Non-relativistic bending limit:

$$E_k = \frac{p^2}{2m} = \frac{(B\rho q)^2}{2m} = \frac{Q^2}{A} \frac{(B\rho e)^2}{2u} = \frac{Q^2}{A} K_b$$

Example:

$$\rho$$
 = 1 m

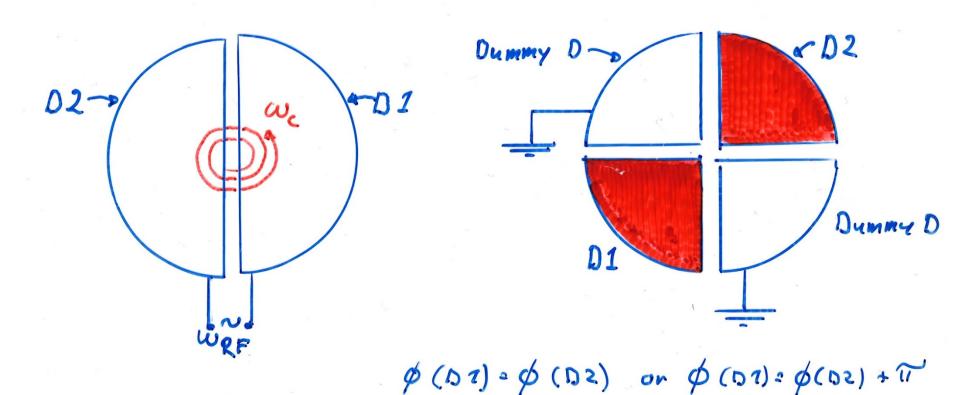
$$B = 1.7 T$$

$$K_{b} = 139 \text{ MeV}$$

Also

harmonic number

h=1,2,3, ...

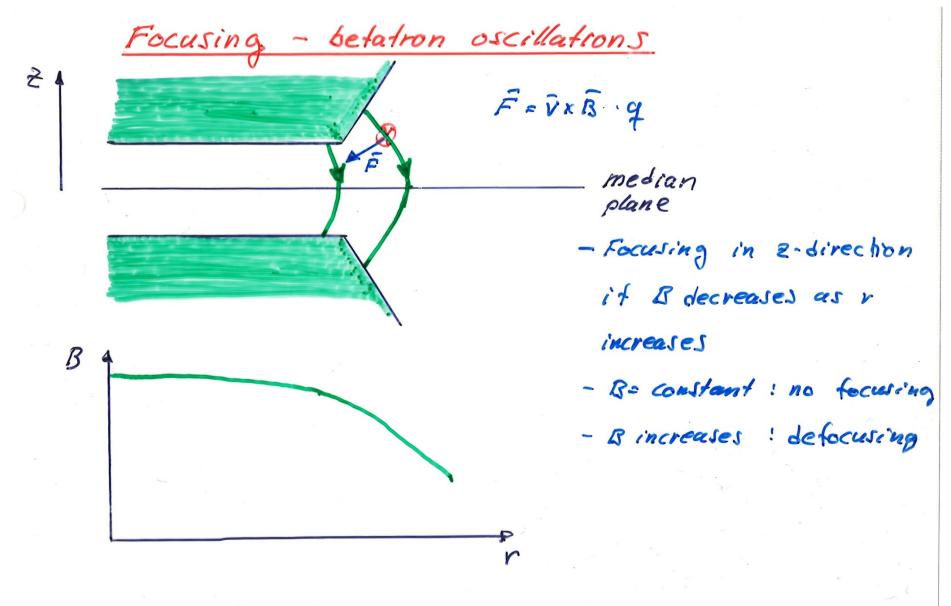


· polarity of electrode changes when the particle is inside Dee — p new acceleration in next gap compare with Videroe

$$= \sum_{n=1}^{\infty} \frac{p^2}{2m} = \frac{(89)^2 e^2}{2m} \approx 20 \text{ MeV}$$

~ not possible in a classical cycl.

Energy limit in classical cyclotrons



Let's look at the tocusing conditions in more dedoil,

Kerst-Serber equations

Consider a moving particle in a classical cyclotron $B \neq B(B)$, B = (r, z)near equilibrium orbit. ($E \circ 0$)

On $E \circ 0$ (bending radius = $g \circ 0$) $mv^2 = g \circ 0 \circ 0$ ($g \circ 0 \circ 0$)

Write: $g \circ 0 \circ 0 \circ 0$ Write: $g \circ 0 \circ 0 \circ 0$ $g \circ 0 \circ 0 \circ 0$

10 Radial focusing

$$m\ddot{v} = -qBv + m\frac{v^2}{v}$$

$$m\ddot{x} = -qVB_0\left(1+k\frac{k}{S}\right) + mv^2\left(S+x\right)^{-1}$$

$$= -qVB_0\left(1+k\frac{k}{S}\right) + mv^2\frac{1-\frac{k}{S}}{S}$$

$$= -qVB_0 - qVB_0k\frac{k}{S} + m\frac{v^2}{S} - m\frac{v^2}{S}\frac{k}{S}$$

$$= -qVB_0k\frac{k}{S} - qVB_0\frac{k}{S}$$

(a)
$$x' + (w_c^2 k + w_c^2) x = 0$$

 $x' + (w_c^2 (1+k) x = 0$

2° Axial focusing

=P Z= Az cos (V-k wct + do)

limited if k <0

FOCUSING IN BOTH PLANES IF

Note! Often in the literature -k=n= B -r B Jr

n, k field index

Vr periods /turn radially

V2 - " axially

Synchro cyclotron

Ex increases — om increases — owe decreases

(we decreases also due to focusing condition)

- Decrease accelerating frequency with increasing energy = synchrocyclotron

ADVANTA GES

- + higher energy
- + possibility for better axial focusing

DISADUANTAGES

- only one (few) pulse at the time can be accelerated — intensity goes down

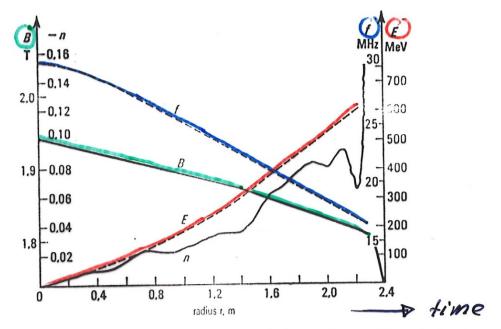
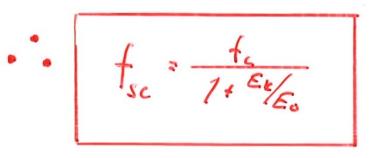
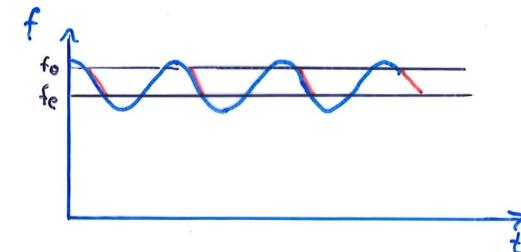


Figure 3.7 Parameters of 600-MeV CERN synchrocyclotron (B— induction, E— proton energy, f—accelerating-voltage frequency, n field index)

Synchro cyclotron frequency





frequency modulated

= acceleration of one pulse

U = 10-30 kV

Prepetition rate / cycling rate = 50 - 500 Hz

Isochronous cyclotron

- = sector focusing cyclotron
- = AVF cyclotron (Azimuskally Varying Field)

Another way to compensate for the mass increase or frequency decrease is to increase magnetic field with radius (energy)

BUT

Kerst-Serber: avial defocusing

- axial focusing must be increased by modifying the magnetic field so that the synchronous condition is fulfilled
 - · cannot be done radially (syach. condition)

P Question: Can axial tocusing be increased

modifying the field azimuthally so that

(By corresponds to synchronous field?

Answer: YES

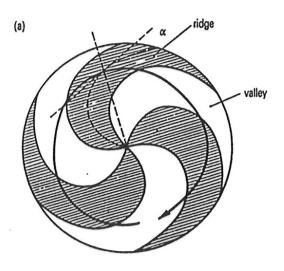
Try sectors and examine the components of Lorenz torce

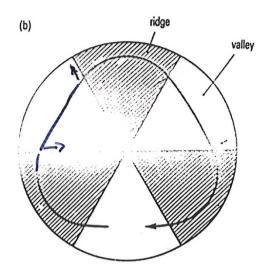
Primary motion Va

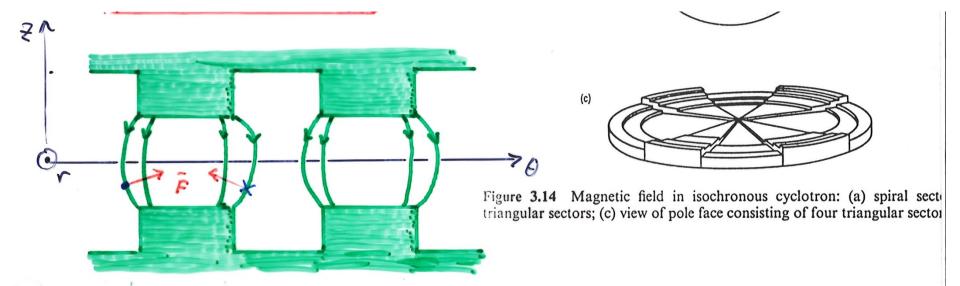
at the sector edge

Themas focusing

ISOCHRONOUS CYCLOTRONS







Fz = q Vr Bo = Thomas force

ALWAYS towards the median plane

... axial focusing

Spiral effect

```
At the sector edge (z to) Br to

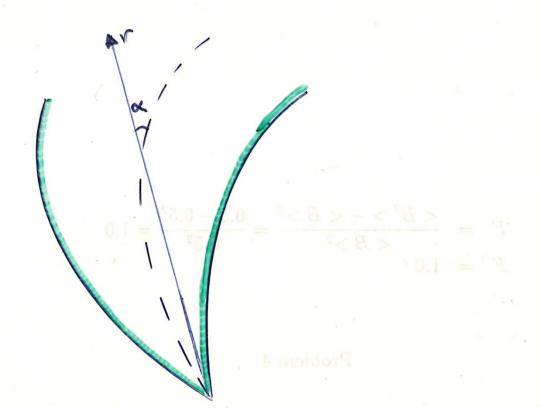
Into hill (sector) Br 70 \
Out from hill Br <0)
```

- FOCUSING - DEFOCUSING - FOCUSING - DEFOCUSING - ...
Totally, FOCUSING (compare with light optics)

50:

"Arral focusing that was lost due to the Mochronous condition was gained back with (spiral) sectors"

Spiral angle &



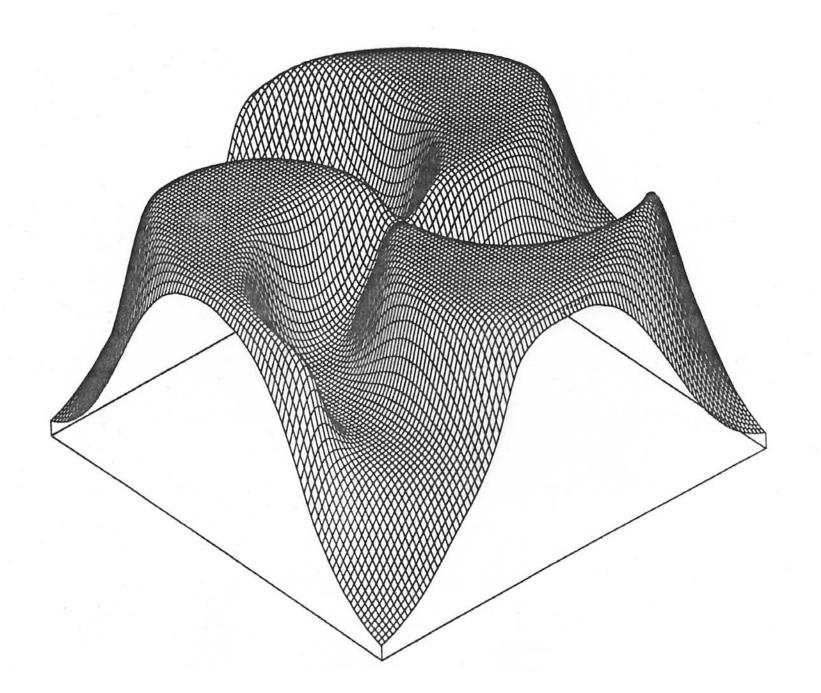
Define FLUTTER F:

N= number of sectors

3 sector cyclotron:

Note!

Adding sectors decreases flutter



Synchronous condition:

$$B = \frac{m}{q} w_c$$

$$=\frac{B_0}{\sqrt{1-\left(\frac{y}{c}\right)^2}}$$

Field shape (radially)?

$$K = \frac{r}{B} \frac{dB}{dr}$$

$$= \frac{rB_0}{B} \frac{d}{dr} \left(1 - \frac{\omega^2}{c^2} r^2\right)^{-\frac{2}{2}}$$

field index corresponding to isochronous field

$$V_{2}^{2} = -K + F(1 + 2 + am^{2} x)$$

$$= 1 - y^{2} + F(1 + 2 + am^{2} x) + 0$$

F(7+2 tom'x) > ye'- 1 Focusing condition

$$\gamma = \frac{E_k + m_0 c^2}{m_0 c^2}$$

For room temperature cyclotrons (B < 2 T)

- Flutter F does not depend on B
 - So, maximum γ (v or E/A) limited by magnet geometry

For superconducting cyclotrons (B >> 2 T) iron is saturated

- $\langle B^2 \rangle \langle B \rangle^2 = constant$ (given by sector geometry)
 - Hence, Flutter decreases as 1/B²
 - Focusing limit:

$$\frac{E}{A} = K_f \frac{Q^2}{A^2}$$

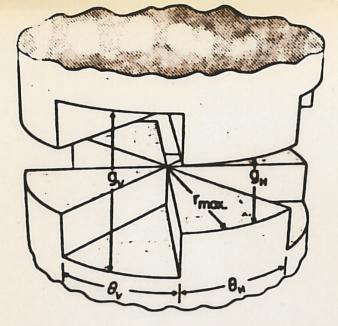


Fig. 2. Schematic drawing of the pole tip geometry assumed in our calculations. The hillgap "gh" and the valley gap "gv" are excrywhere uniform. Likewise the hill angular width " θ_h " and the valley ang far width θ_v " are indent of radius although in species the ingular section of the hill edge varies with radius (sect. 4). The sector number is given by $h = 360 / (\theta_h + \theta_v)$. The pole outer radius is designated " r_{max} ".

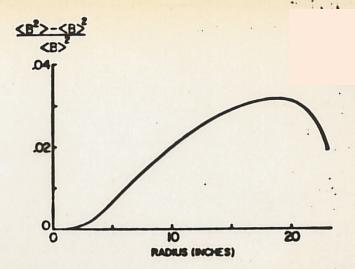


Fig. 3. Flutter [eq. (1)] vs radius for the "standard case" pole tip, which has $\theta_h = \theta_v = 45^\circ$, $g_h = 3^\circ$, $g_v = 36^\circ$, $r_{max} = 24^\circ$ and $\sim B^\sim$ as in eq. (3) (≈ 3.5 T). The focusing is adequate for about $\sim B^\sim$ MeV/nucleon.

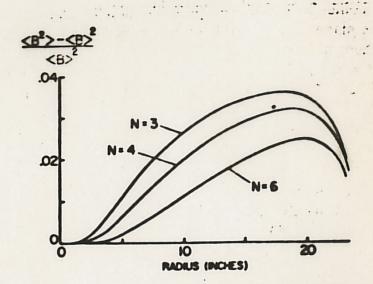


Fig. 4. Flutter vs radius for the standard four-sector case, compared with values for three sectors and six sectors. In all cases $\theta_b = \theta_v$ and other parameters are the standard case values.

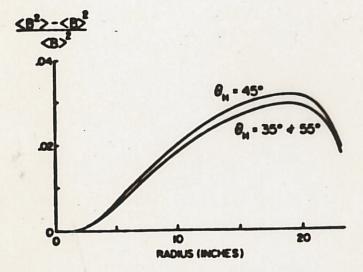


Fig. 5. Flutter vs radius for the standard case $\theta_h = 45^\circ$, and for narrower and wider hills, $\theta_h = 35^\circ$ and 55° (flutter identical). In all cases $\theta_v + \theta_h = 90^\circ$ and other parameters are the standard case values.

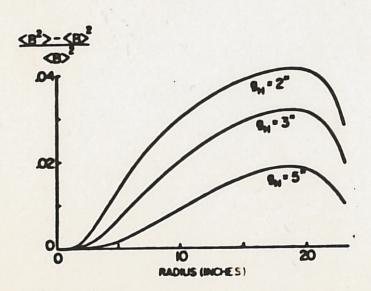


Fig. 6. Flutter vs radius for $\frac{1}{2}$ standard case, $\frac{1}{2}$ = 3°, and for smaller and larger hill gaps, $\frac{1}{2}$ = 2° and $\frac{1}{2}$ = 5°. An other parameters are the standard case values.

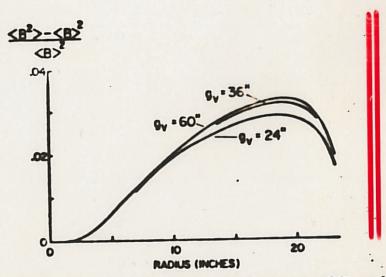


Fig. 7. Flutter vs radius for the standard case, $g_v = 36^\circ$, and for smaller and larger valley gaps $g_v = 24^\circ$ and $g_v = 60^\circ$. All other parameters are the standard case values.

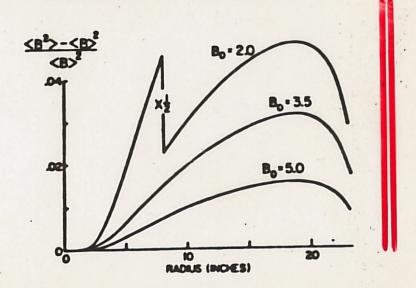
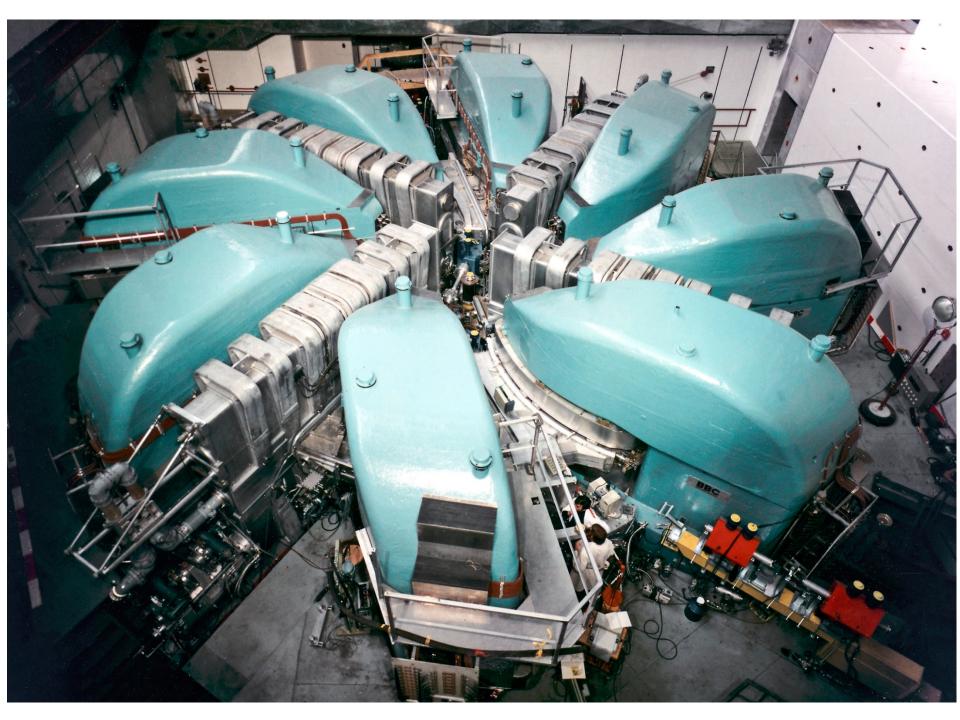


Fig. 8. Flutter vs radius with the central field at the 3.5 T standard case value, and lowered and raised to 2.0 T and 5.0 T.

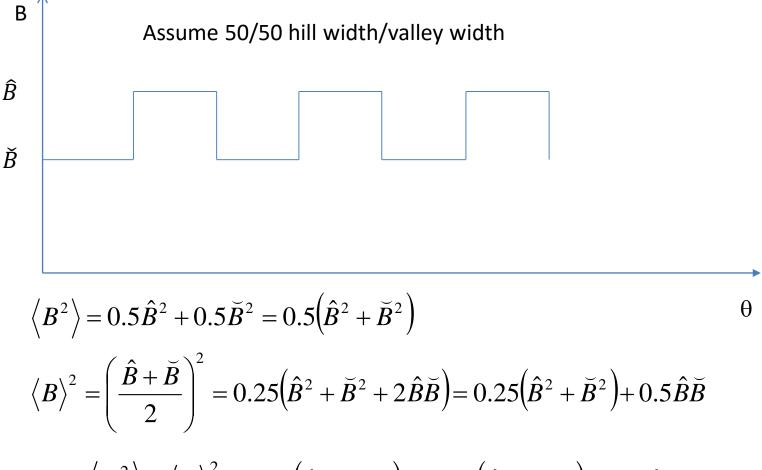
All other parameters are the standard case the peak value for the 2.0 curve is 0.05.

Separated sector cyclotrons

- For higher energy light ions, axial focusing sets the limit
 - Increase spiral angle
 - Increase flutter F
 - Zero field in the valleys
 - Separated sectors
 - » Space for equipment between the sectors (dipoles)
 - Effective resonators for high accelerating field



Flutter



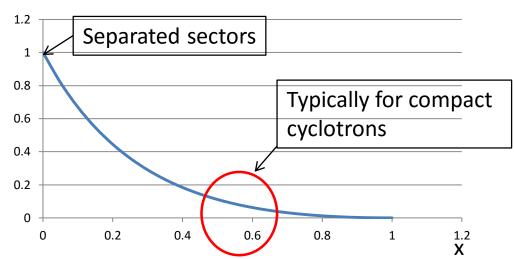
$$F = \frac{\langle B^2 \rangle - \langle B \rangle^2}{\langle B \rangle^2} = \frac{0.5(\hat{B}^2 + \breve{B}^2) - 0.25(\hat{B}^2 + \breve{B}^2) - 0.5\hat{B}\breve{B}}{0.25(\hat{B} + \breve{B})^2}$$

$$= \frac{0.25(\hat{B}^2 + \breve{B}^2) - 0.5\hat{B}\breve{B}}{0.25(\hat{B} + \breve{B})^2} = \frac{\hat{B}^2 + \breve{B}^2 - 2\hat{B}\breve{B}}{(\hat{B} + \breve{B})^2}$$

$$= \frac{(\hat{B} - \breve{B})^2}{(\hat{B} + \breve{B})^2}$$
Flutter
$$\breve{B} = x\hat{B}$$
Separated sectors

Tunically for confidence in the sector of t

$$F = \frac{\left(1 - x\right)^2}{\left(1 + x\right)^2}$$



Remember: $F(1+2\tan^2\alpha) > \gamma^2-1$



For high E/A, choose separated sectors

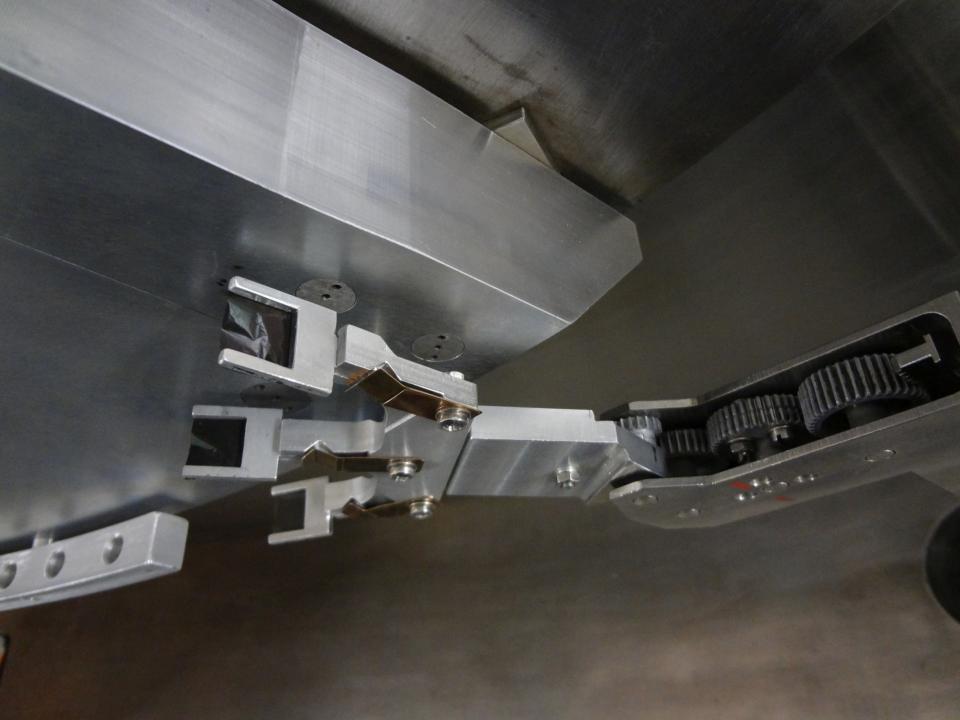




Spiral inflector



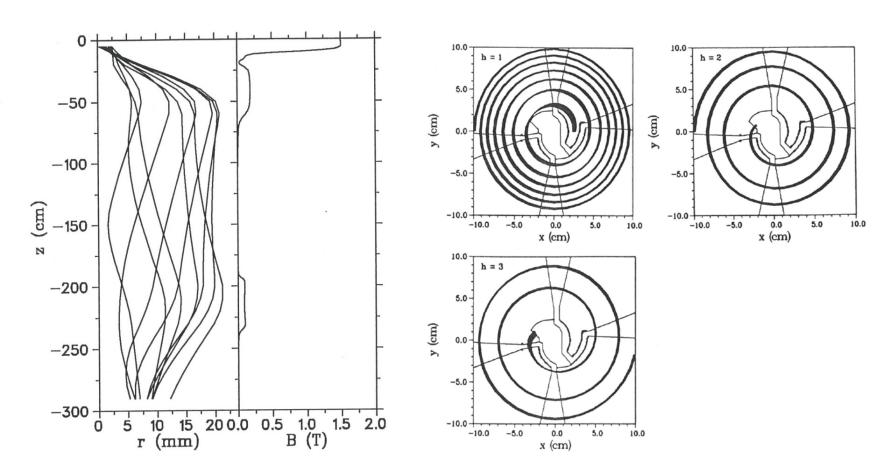




Injection/central region and extraction

Injection

- External ion source
- Matching the beam into the cyclotron's
 - Central region acceptance
 - Accelerated equilibrium orbit "eigen ellipses"
- Low-energy beam
 - Possible space charge limitation



Forces in the cyclotron

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

Typically
$$\hat{E} \cong 10 \text{ MV/m}$$

$$B \cong 1.5 \text{ T}$$

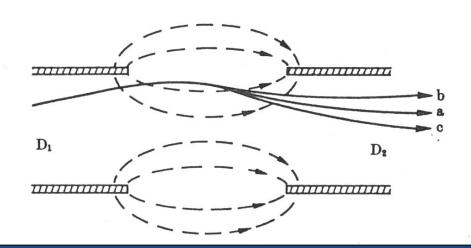
$$F_E = F_B$$

$$\Rightarrow v \cong 0.02 c$$

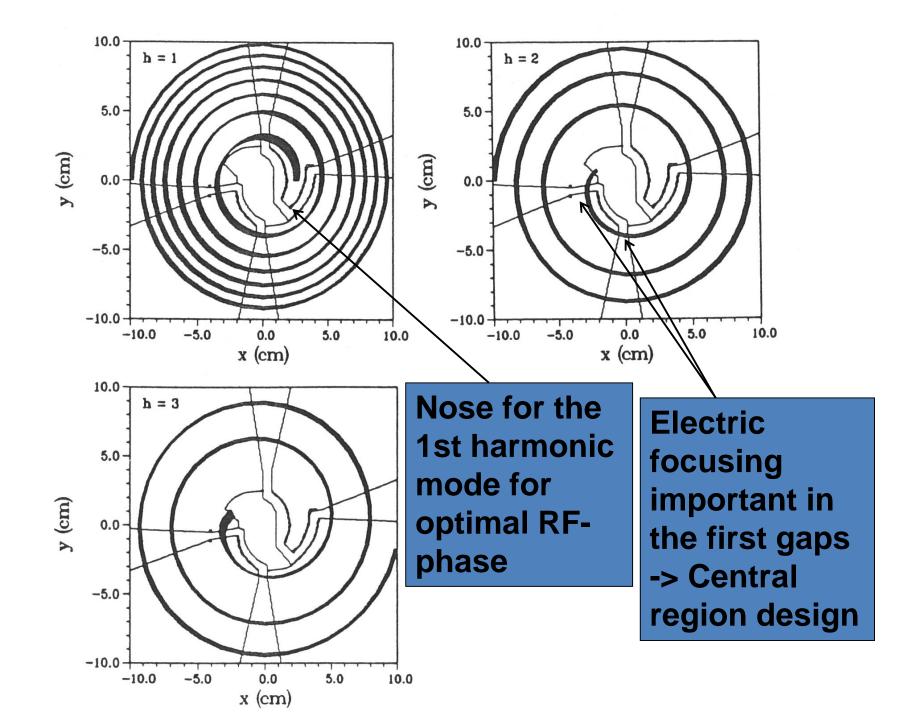
$$\Rightarrow \frac{E}{A} \cong 200 \text{ keV/n}$$

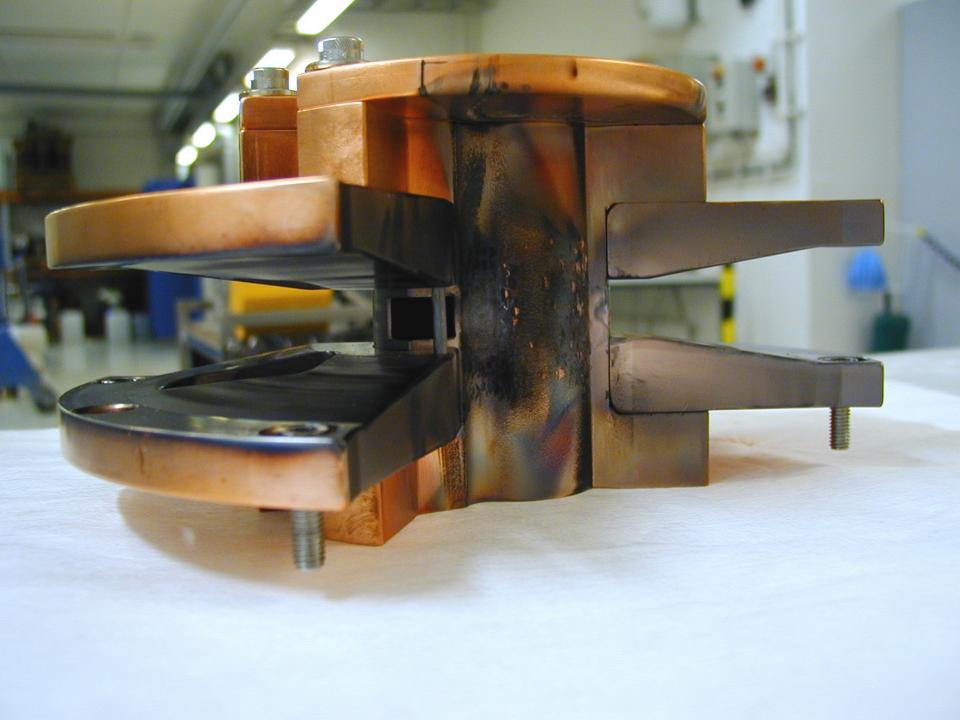
This energy is reached during 1 – 2 turns

Outside the central region only magnetic forces (bending, focusing) are relevant. However, electric focusing is important along the first 1 – 2 turns.

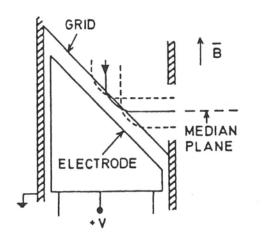


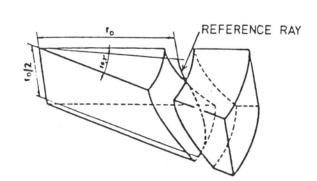
Transit time effect in an accelerating gap in a) static field, b) increasing field and c) decreasing field. The effect is exaggerated

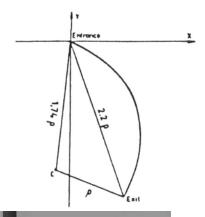




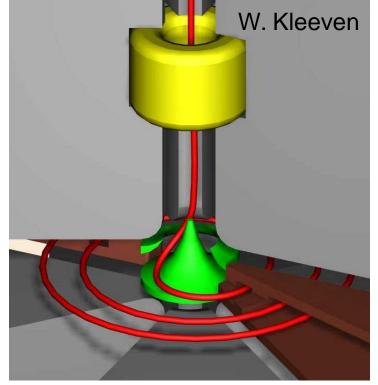
Inflectors





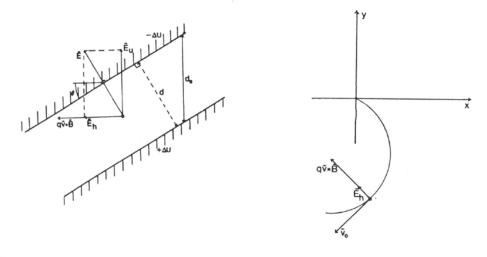






Spiral inflector

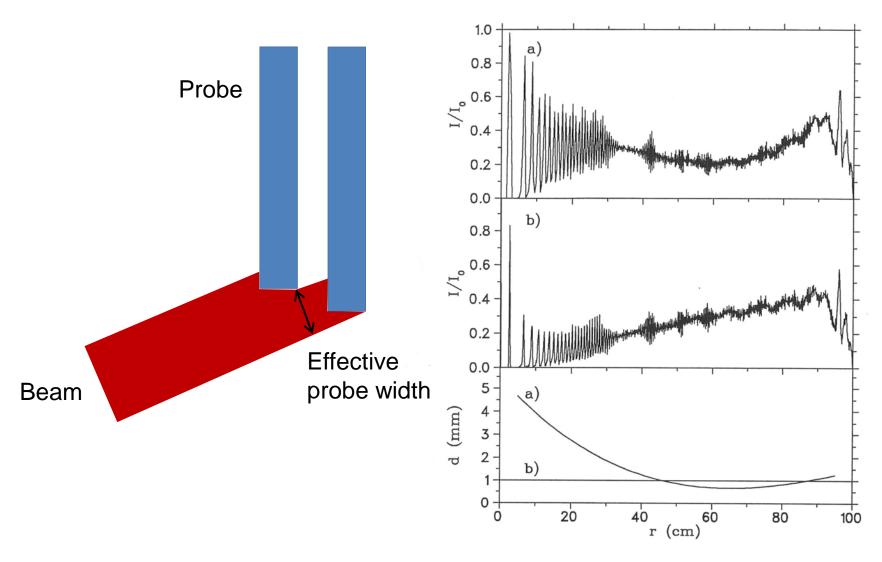
Beam bending without magnetic field



a) Cross-section of the spiral electrodes and b) beam projection on xy plane

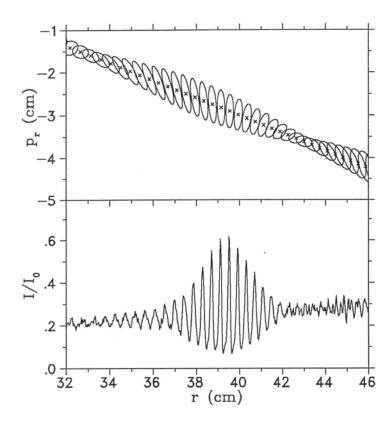


Injection has an effect on beam behaviour in the cyclotron

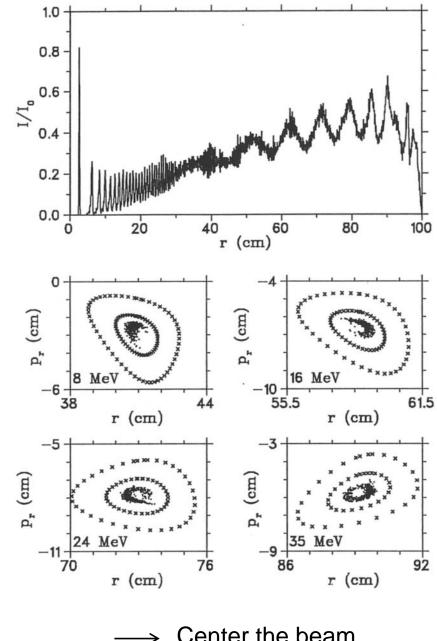


Differential probe scan with a) a changing effective probe width and b) with a constant effective probe width.

The beam rotates at the radial betatron frequency



Match the beam into the acclerated equilibrium orbit eigen ellipses with quadrupoles (4)



Center the beam



Classification of extraction schemes



Circular accelerators

No extraction problem

Constant orbit radius (sychrotrons, betatrons)

Increasing orbit radius (cyclotrons, synchrocyclotrons)

Pulsed electromagnetic fields (Kickers) Resonant (slow) extraction $v_r = N/3$

Stripping by foil (e.g. H⁻)

Electromagnetic fields

Integer resonance

$$v_r = N$$

Half integer resonance

$$v_r = N/2$$

(regenerative extraction)

Brute force extraction

Precessional extraction

Extraction by acceleration

Radial increase of the orbit

- By acceleration
- By magnetic pumps

$$\frac{dR}{dn} = \frac{dR}{dn} (accel) + \frac{dR}{dn} (magn)$$

$$\frac{dR}{dn} \text{(accel)} = R \frac{E_g}{E} \frac{\gamma}{\gamma + 1} \frac{1}{v_r^2}$$

Three ways to get a high extraction rate

- 1. Build cyclotrons with a large average radius (without increasing the maximum energy)
- 2. Make the energy gain per turn as high as possible
- 3. Accelerate the beam into the fringe field, where v_r drops

This also calls for high energy gain, since phase slip in the fringe field must be kept small

Item 1. Remember that for the same maximum field and the same energy gain per turn

$$\frac{dR}{dn}$$
 (accel) $\propto \frac{1}{R}$

Item 3. especially important for high energy cyclotrons

$$V_r \approx \gamma$$

Remember: for an isochronous field

$$k = \frac{r}{B} \frac{dB}{dr}$$
 Field index
$$= \gamma^2 - 1$$

And e.g. for a 3-sector magnet

$$v_r^2 \approx 1 + k + 0.675 F (1 + \tan^2 \alpha) + \dots$$

So, e.g. for the PSI 580 MeV cyclotron in the isochronous extraction region

$$\nu_r=1.6$$
 and at the extraction in the fringe field $\nu_r=1.1$ Factor of 2 in turn separation

Resonant extraction

Normally the radial gain per turn by acceleration in not enough

•Magnetic perturbations to enhance the turn separation

The integer resonance $v_r = N$

Brute force

Bump in the axial field
$$\Delta B(r,\theta) = b_N \cos N(\theta - \theta_N)$$

 v_r close to $N \longrightarrow$

The beam is driven off centre, maximum additional radial gain per turn being

$$\frac{dR}{dn} \text{(brute force)} = \pi R \frac{b_N}{NB_0}$$

For a typical conventional cyclotron (B_0 =1.7 T) a bump of 0.1 mT introduces a radial gain of about 0.2 mm!!

To get a desired turn separation bigger bumps are needed

- •"Brute force"
- •This method has been used for example in the AEG compact cyclotron

Precessional extraction

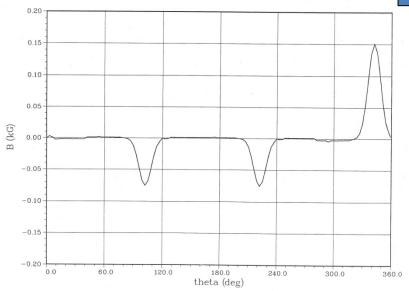
The beam goes through $v_r=1$ resonance with a first order perturbation

•Beam starts to oscillate around its equilibrium orbit with a frequency

$$|\nu_r - 1|$$

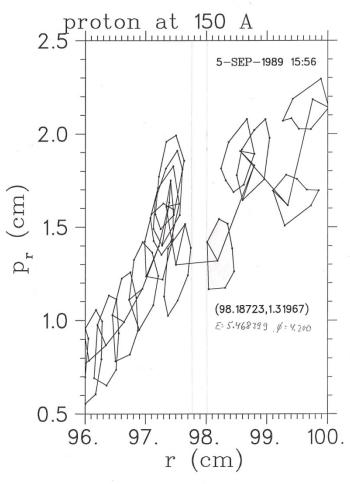
- · v_r decreases with radius
 - •Two consecutive turns oscillate with a slightly different frequency
 - Phase difference between the turns increases
 - •Turn separation increases

Bump with a harmonic coil <B_{harm}>=0

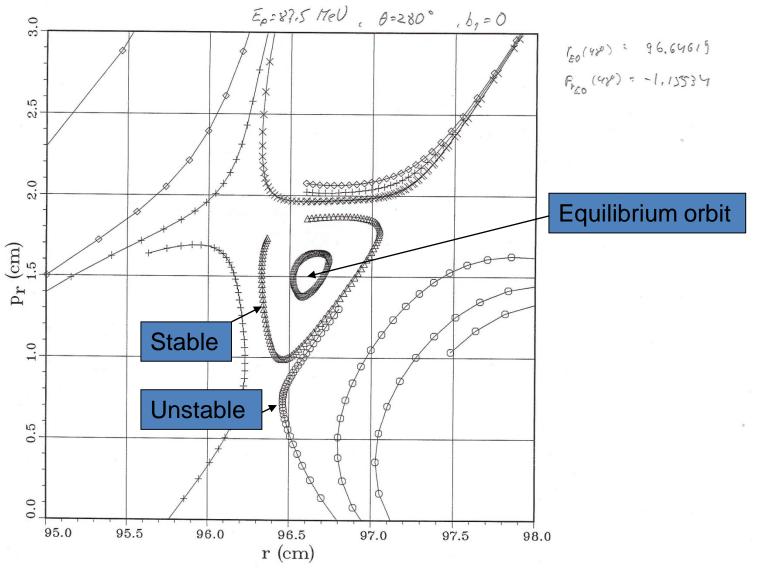


Contribution of harmonic coils in three valleys

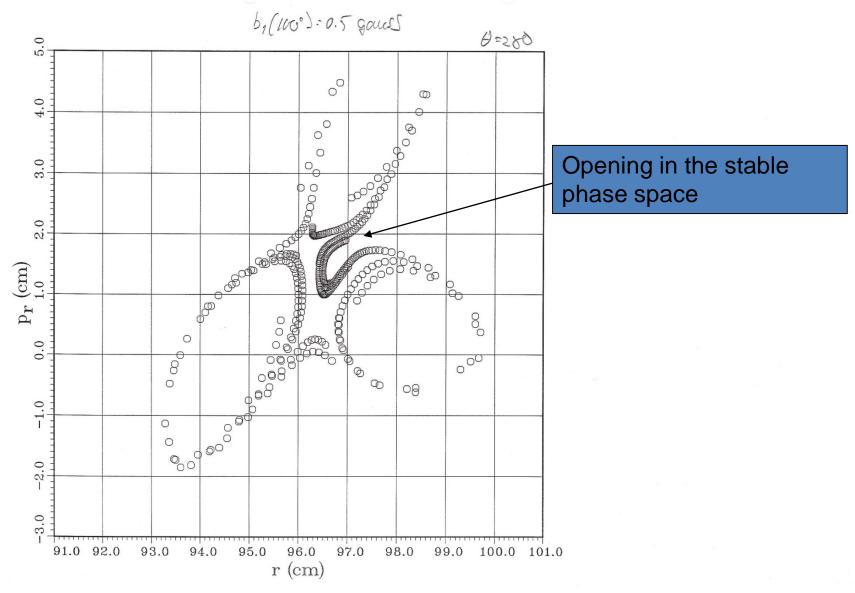




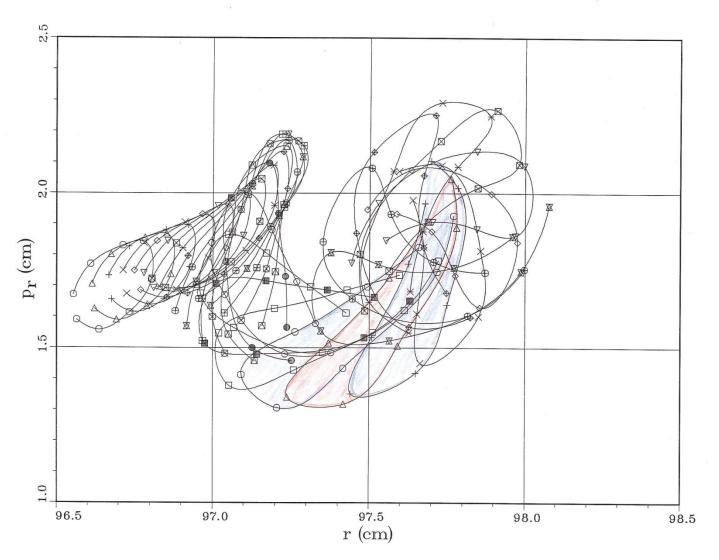
Precession after $v_r=1$ resonance



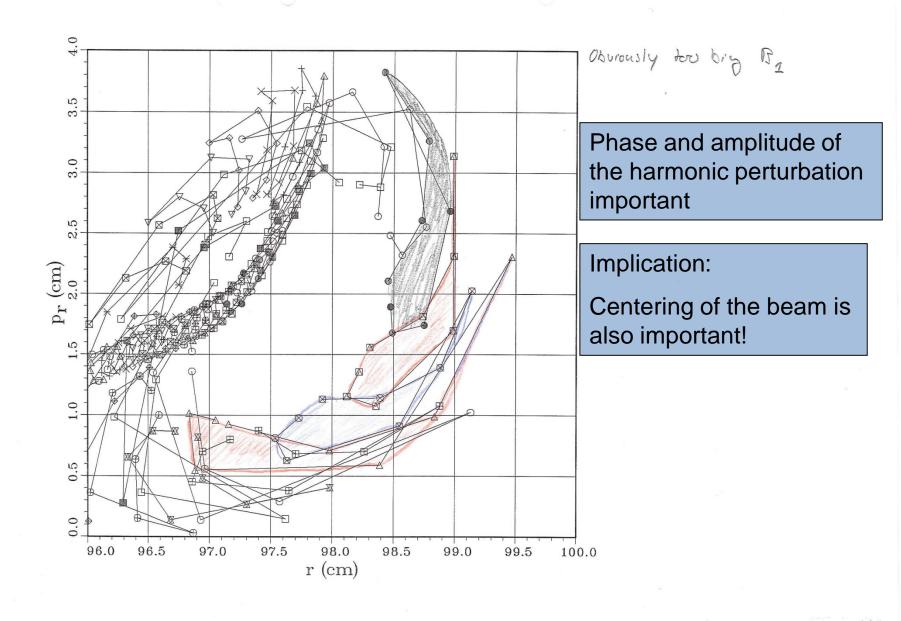
Radial phase space without a 1st order perturbation

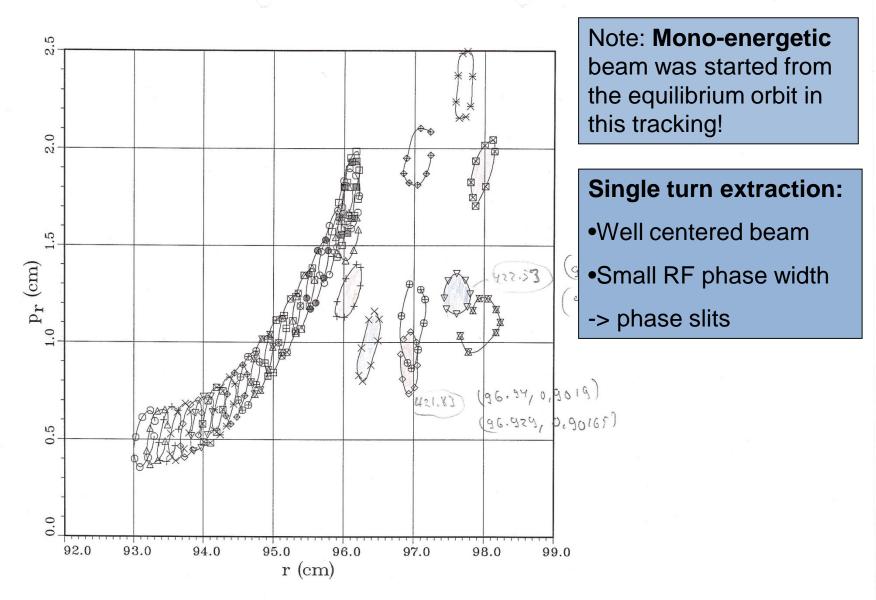


Radial Phase space with a 1st order perturbation



Phase and amplitude of the perturbation is important!



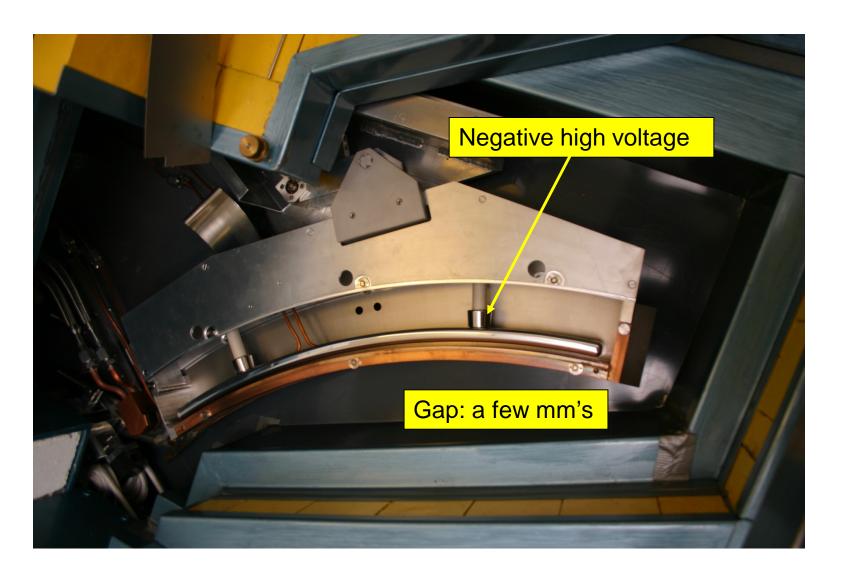


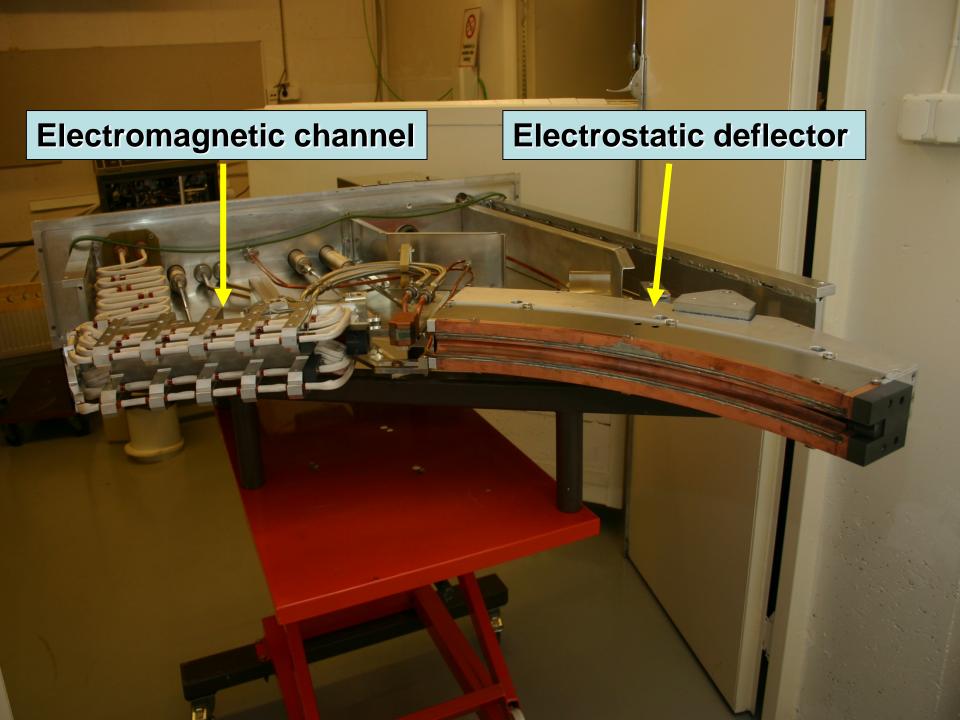
Nice behavior with a proper 1st harmonic perturbation

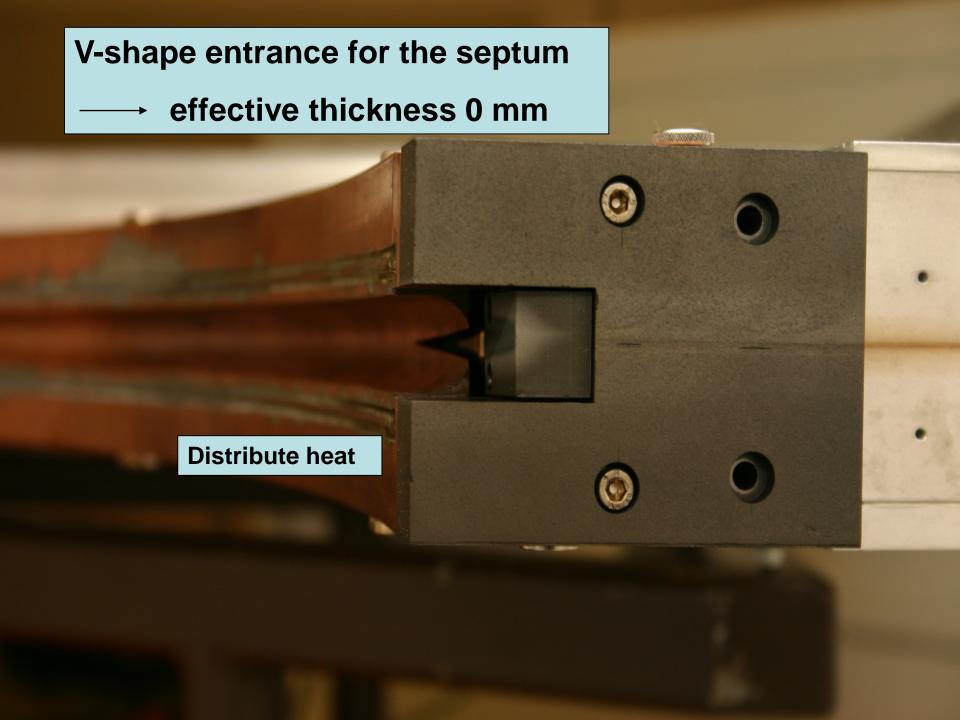
Extraction elements

- (Harmonic coils)
- Electrostatic deflector
- Electromagnetic channel
- (Passive) focusing channels
- Stripper

Electrostatic deflector





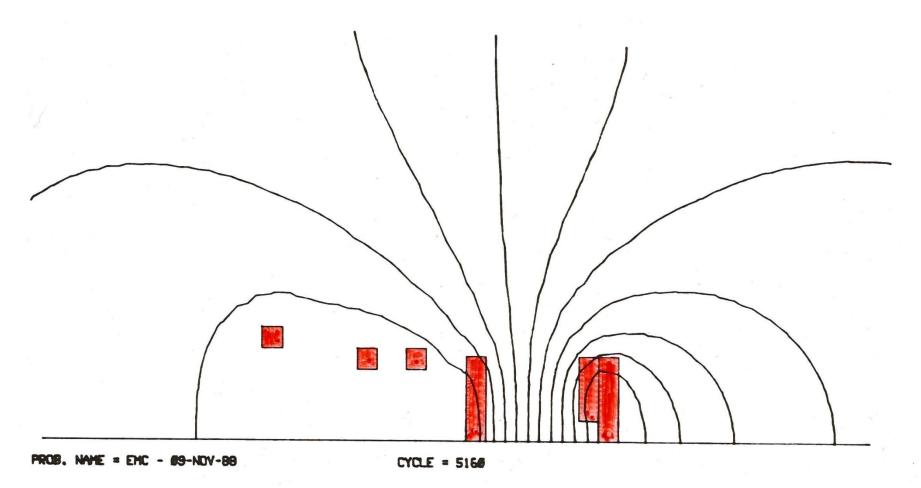


Electromagnetic channel

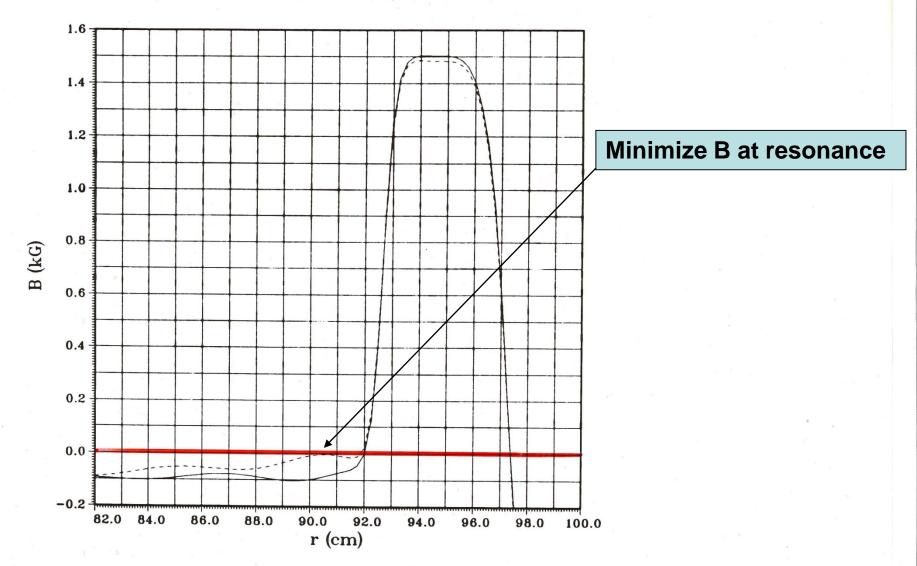


High current in the EMC coil

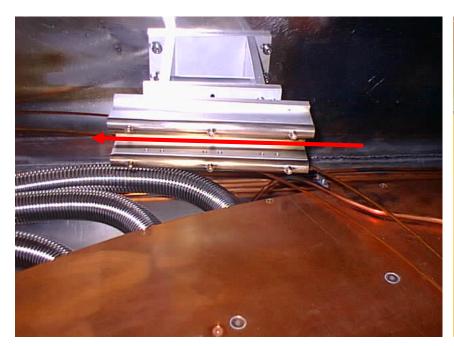
•Main coil current + booster current



EMC field solid original (C60) dashed = #I(-20mm), #2(-2.5mm)



Passive focusing channel





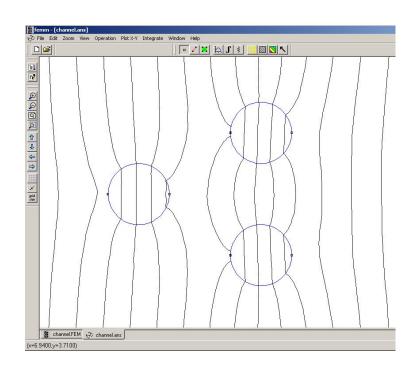
Vertically focusing

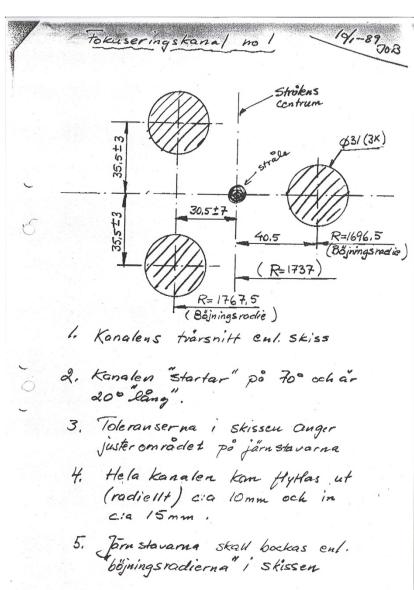
Horizontally focusing

Iron bars are magnetized by the cyclotron magnetic field

Focusing channel

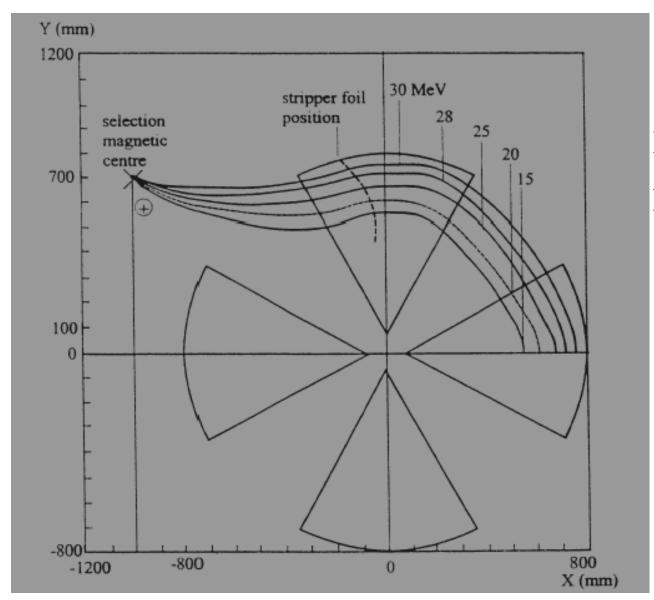
- Extracted beam travels in the fast decreasing fringe field
 - Horizontally defocusing
 - More focusing by shaping the field (gradient) by passive channels





Stripping extraction

- The extraction efficiency for deflector + EMC is typically 50 – 90 %.
 - For high intensities activation, vacuum and melting problems
- For negative ions (H⁻, d⁻) stripping
 - $-1-2 \mu m$ carbon foil strips both electrons away
 - Charge state -1 -> +1
 - Efficiency close to 100 %
 - Short distance in the fringing field
 - Less focusing problems
- Caution! Electromagnetic stripping at high B and high velocity
 - Electron affinity (binding energy) for H⁻ is 0.75 eV



IBA Cyclone 30
Extraction by stripping

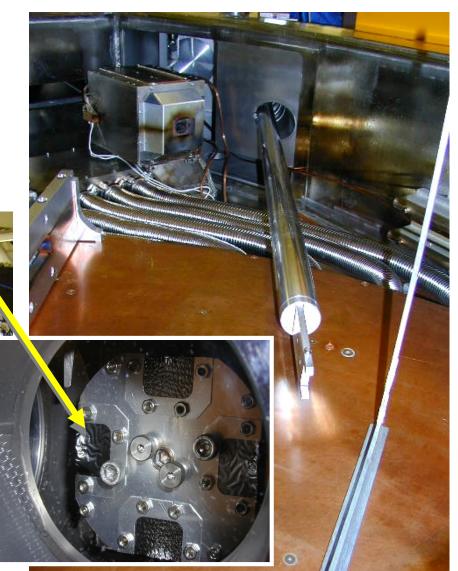
All energies go to one crossover point by proper foil azimuthal position

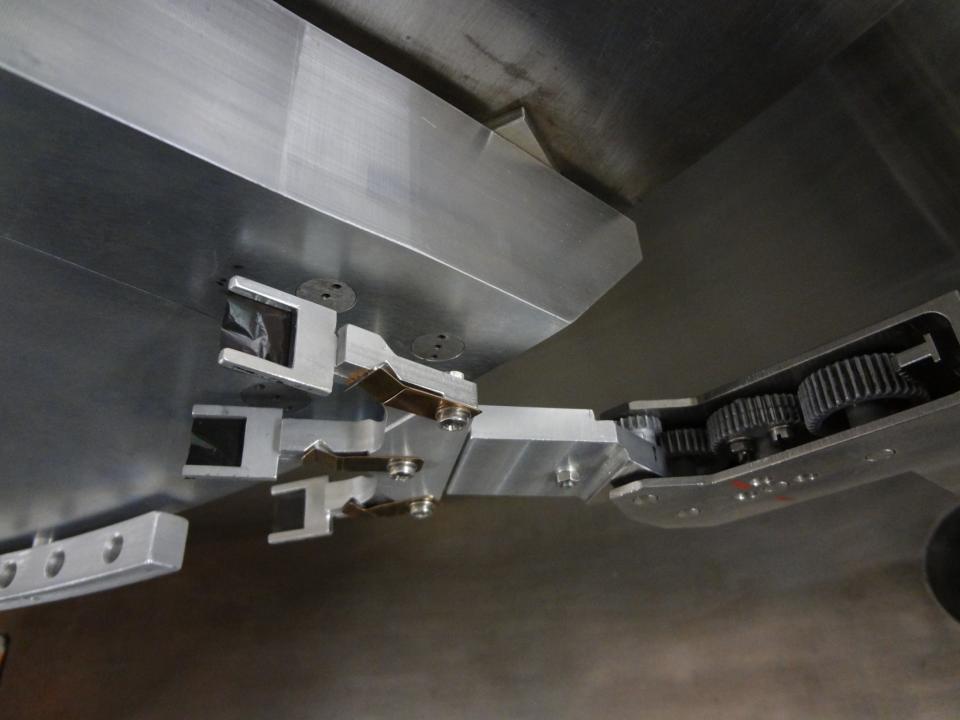
Place combination magnet at crossover

Stripper

- •4 carbon foils (16x22 mm)
- •thickness 1 and 2 μm







Stripping extraction for heavy ions

- Typically $q_2 = 2q_1 (1.4 4)$
 - Initially moderate charge state
 - Limits the maximum energy
 - Motivation
 - High extraction efficiency
 - Naked ion after stripping (no charge distribution)
 - e.g. 300 AMeV Cyclotron proposal (INFN)
 - Easy
 - If not fully stripped then a distribution
 - » Only one charge state has the right trajectory
 - e.g. Dubna

