Nordic Particle Accelerator School 2017

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Introduction to Accelerator Physics

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Outline

- Basic relations (units, kinetic energy, relativistic particles)
- Lorentz force & Maxwell's equations (electrodynamics)
- Different types of accelerators and electron guns
- Oscillating EM fields→linacs
- Circular accelerators
- Synchrotrons and phase stability
- Magnets (dipoles, quadrupoles, …) and focusing properties
- RF cavities and power lost per turn

Basic relations

- Electric charge: electron= -1, proton= +1… $e = 1.6 \cdot 10^{-19} C$
- Energy: electron volts (eV), 1 eV is the energy gained by an elementary charge when is accelerated by a voltage of 1 V.
  - We use: **keV=10³ eV**, **MeV=10⁶ eV**, **GeV=10⁹ eV**, TeV=10¹² eV
- The total energy of a particle is the sum of kinetic and rest energy:
  $W = W_0 + W_k$ where $W_0 = m_0 c^2$

electron W_0= 511 keV
proton W_0= 938 MeV

- $W = mc^2 = m_0 \gamma c^2$
- $W_k = W - W_0 = m_0 \gamma c^2 - m_0 c^2 = m_0(\gamma - 1)$

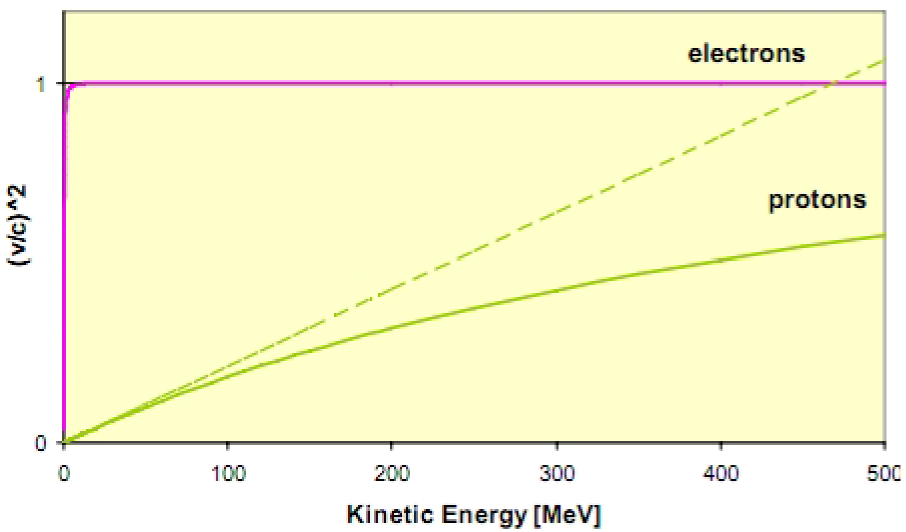
Lorentz factor $$\gamma = \frac{1}{\sqrt{1 - {v^2}/{c^2}}} = \frac{1}{\sqrt{1-\beta^2}}$$

$$v = c\beta$$
velocity

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}}$$

Relativistic particles

$W_k >> W_0$ and $v \approx c$



Example for 1.5 GeV kinetic energy:

electrons, $\gamma = 2940$, $\beta = v/c = 0.999999942$

protons, $\gamma = 2.6$, $\beta = v/c = 0.923$

~5.1 MeV for e^- and
~9.4 GeV for p to get relativistic

$$p = \gamma \beta m_0 c$$

$$E = mc^2 = \sqrt{p^2c^2 + m_0^2c^4}$$

Lorentz force

$$\overrightarrow{F_{\mathcal{L}}} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

$$\begin{cases} \dfrac{d}{dt}(\gamma m_0 \vec{v}) = q\left(\vec{E} + \vec{v} \times \vec{B}\right) \\ \dfrac{d}{dt}(\gamma m_0 c^2) = q\vec{v}\vec{E} \end{cases}$$

Acceleration and steering

Energy gain rate (or loss)

If $v \approx c \quad B = 1T \sim E = 300MV/m$

- Bending: dipole magnets
- Focusing: quadrupole magnets
- Acceleration: electric field
  - the particles are accelerated, i.e., their kinetic energy increases= their momentum increases

Maxwell's equations

Ampere's circuital law

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Maxwell-Faraday's induction law

Gauss' law for magnetism

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \cdot \vec{E} = \frac{1}{\varepsilon_0}\rho$$

Gauss' law

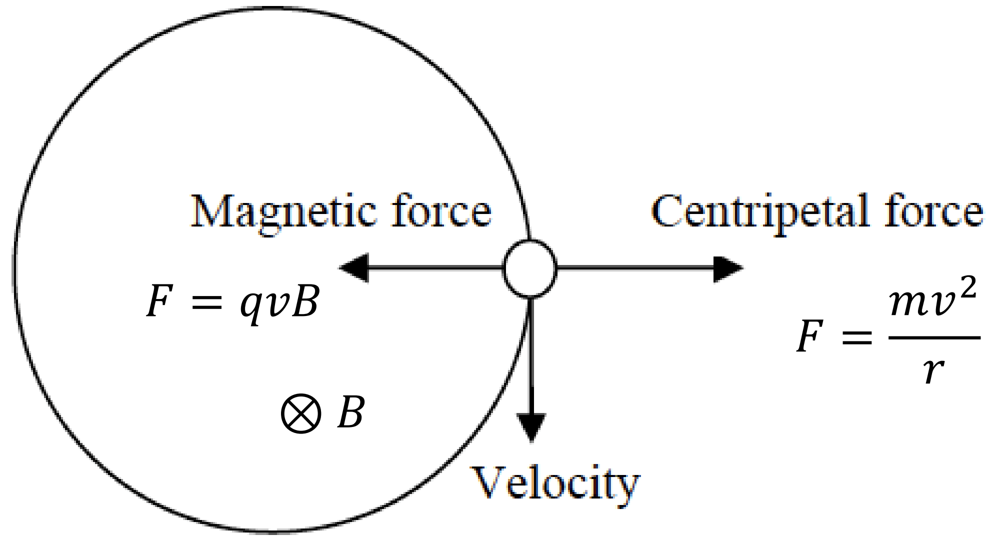
They describe the evolution of electromagnetic fields in time and space

Magnetic field $\vec{\boldsymbol{H}} = \frac{1}{\mu_0}\vec{B}$

Magnetic flux density

Circular motion

Static magnet
No accelerating field
Motion with radius *r*



$$\begin{cases} \vec{F} = \gamma m_0 \frac{d}{dt}(\vec{v}) = q(\vec{v} \times \vec{B}) \\ \frac{d}{dt}(\gamma) = 0 \end{cases}$$

$$evB = \frac{mv^2}{r} \longrightarrow r = \frac{mv}{qB}$$

$$\frac{1}{r} = \frac{eB}{p} = \frac{ceB}{cp} = ce\frac{B}{E}$$

$$\frac{1}{r}[m] \approx 0.3\frac{B[T]}{E[GeV]}$$

Examples:

B, T	E, GeV	r, m
1	4.5	15
1.5	3	6.67
2	27	45

Accelerators zoo

- DC guns
- Electrostatic accelerators (van der Graaf)
- Linacs (Wideroe, Alvarez, Travelling and Standing Waves RF structures)
- Cyclotrons, synchrocyclotrons, isochronous cyclotrons for protons and ions
- Synchrotrons and microtrons for electrons and high relativistic protons and ions

Static **electric fields**

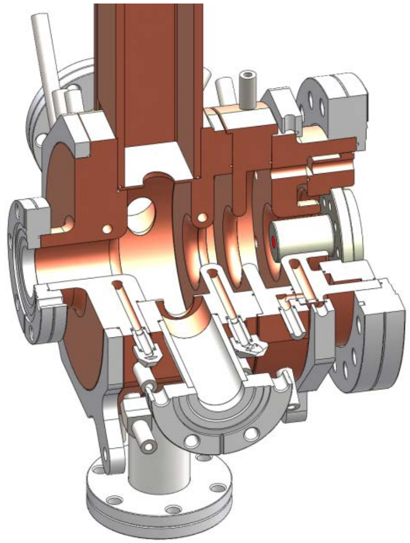
acceleration

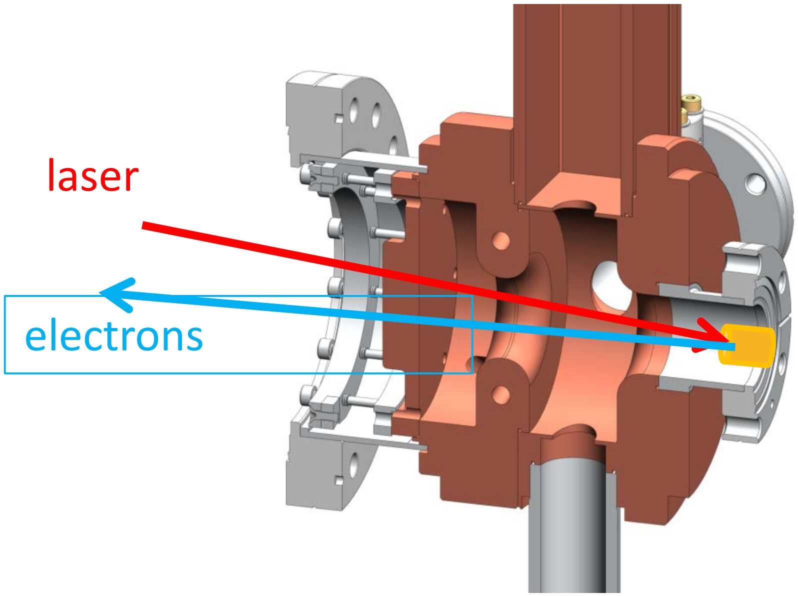
Time varying **electric fields**

Static magnetic or electric fields→guidance/steering

The Betatron is the exception where a time varying magnetic field gives an acceleration of electrons

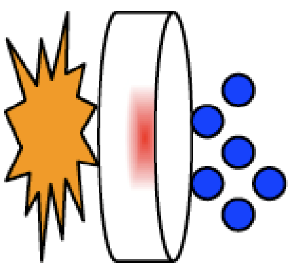
Electrostatic accelerators and DC guns



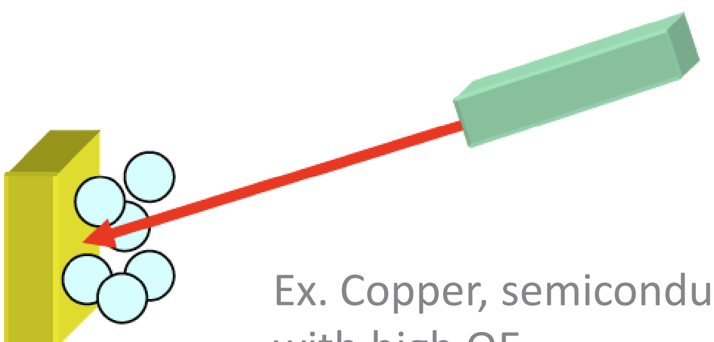


Free electrons can be created:

with a heat in a thermionic cathode or with a light pulse hitting a photocathode



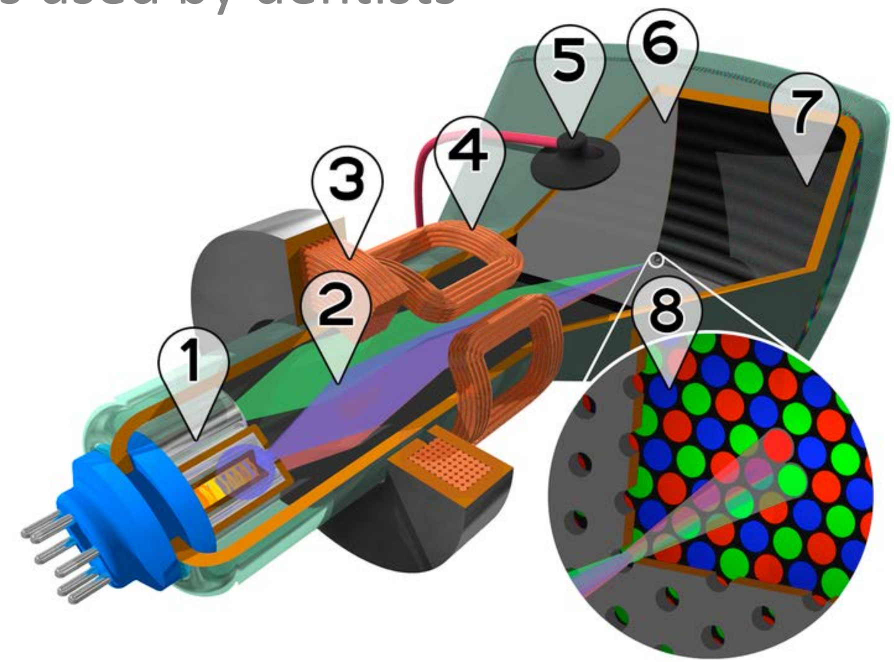
Ex. BaO, T=1000°C



Ex. Copper, semiconductors with high QE

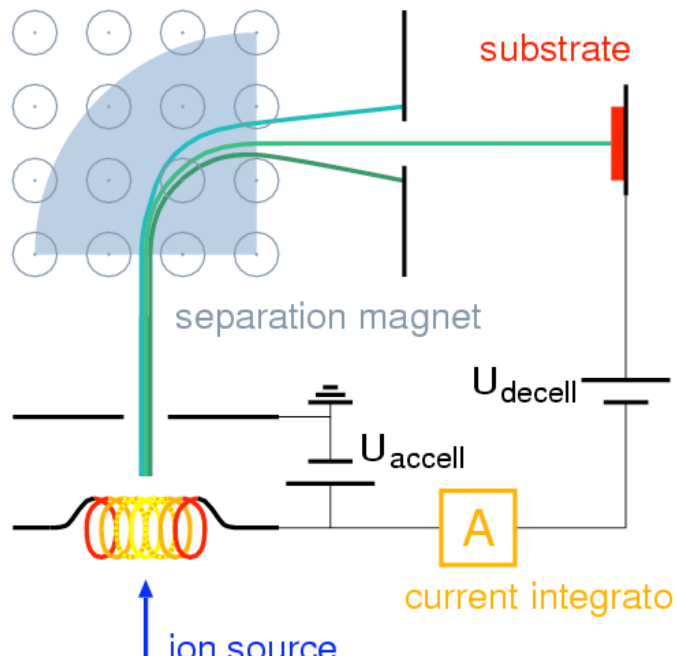
Static electric fields can accelerate

- Old cathode tube TV-sets is an electrostatic accelerator.
- The x-ray equipment that is used by dentists

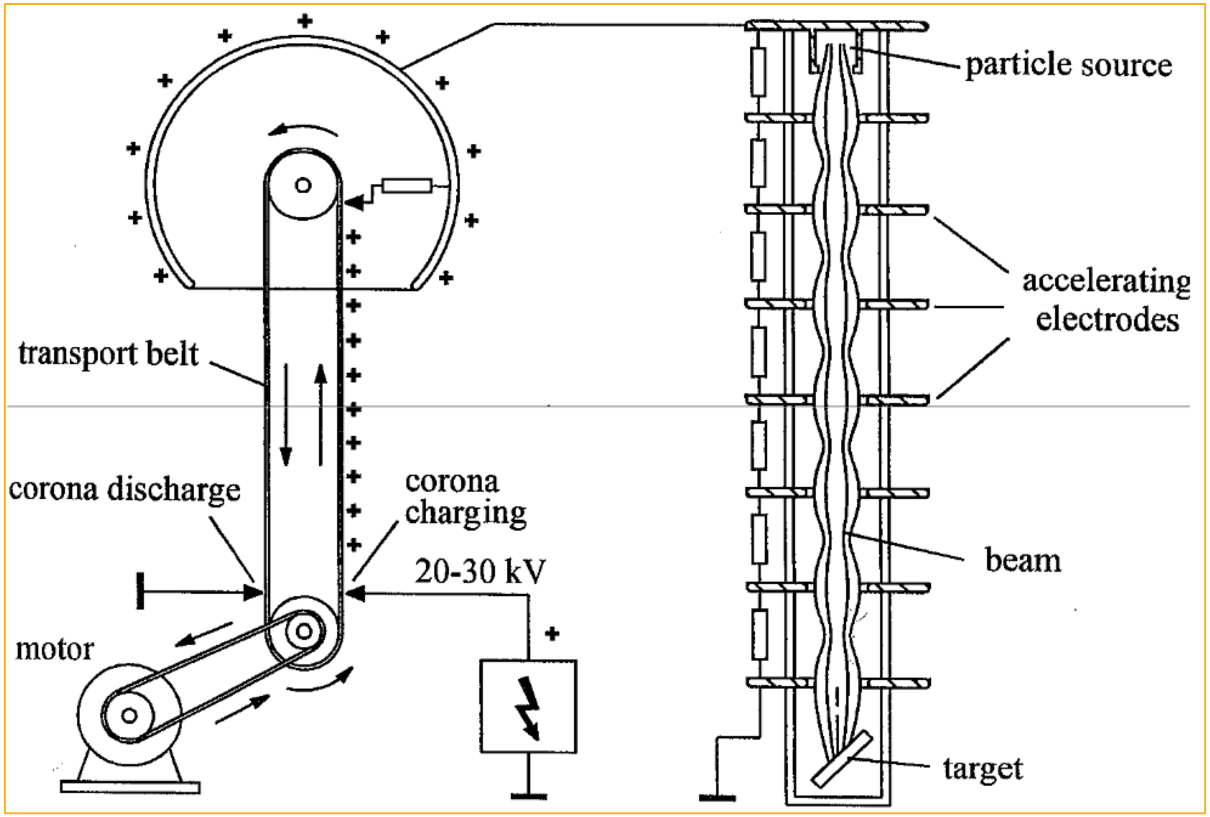


Electrostatic accelerators- Ion implanters

- Ion implanters are used in the semiconductor industry to dope silicon wafers with ions.
- Ion implanters are also used for surface treatment of tools to make them more wear resistant.
- The energy of the ions is typically 10 to 500 keV.



Van de Graaff accelerator



K.Wille 'The physics of Particle Accelerators'

- Van de Graaff generator can reach 2 MV (up to 10MV with SF6)
- Charge from corona formation around a sharp electrode is transferred onto the belt
- Charge is collected on the dome

Different versions exists which are called e.g. Pelletron, Laddertron and Tandem Accelerator

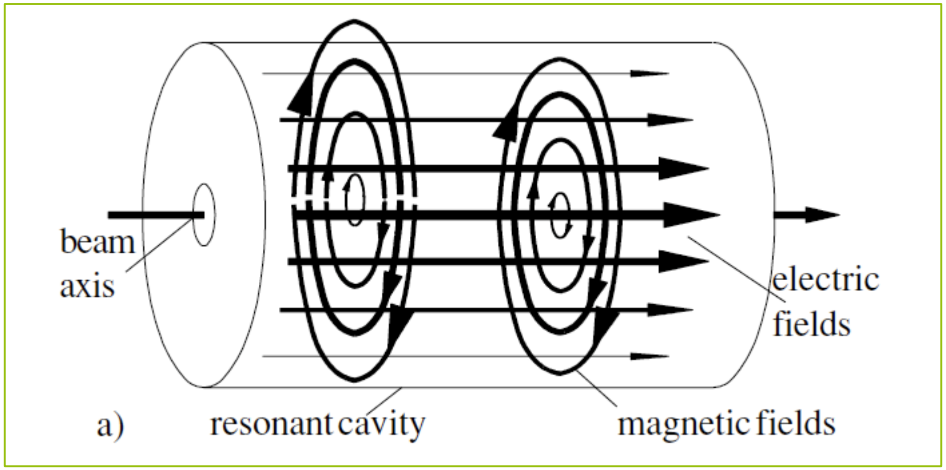
Electrostatic accelerators

- Maximum voltage is about 30 MV which gives a maximum energy of 30 MeV (e- or prot.).
- Electrons becomes relativistic while protons and ions are far from being relativistic.
- Electrostatic accelerators are more common than accelerators using oscillating fields.

Cockroft-Walton accelerator

Oscillating electric fields

- Used to accelerate to high energies
- For higher frequencies, radio waves are trapped in RF cavities having a resonance frequency identical to the radio waves



1924: Gustaf Ising published a concept for the linear accelerator based on oscillating electromagnetic fields

1928: Rolf Widerøe demonstrated it

Linear accelerators(=<u>linac</u>) – one passage through the RF cavities

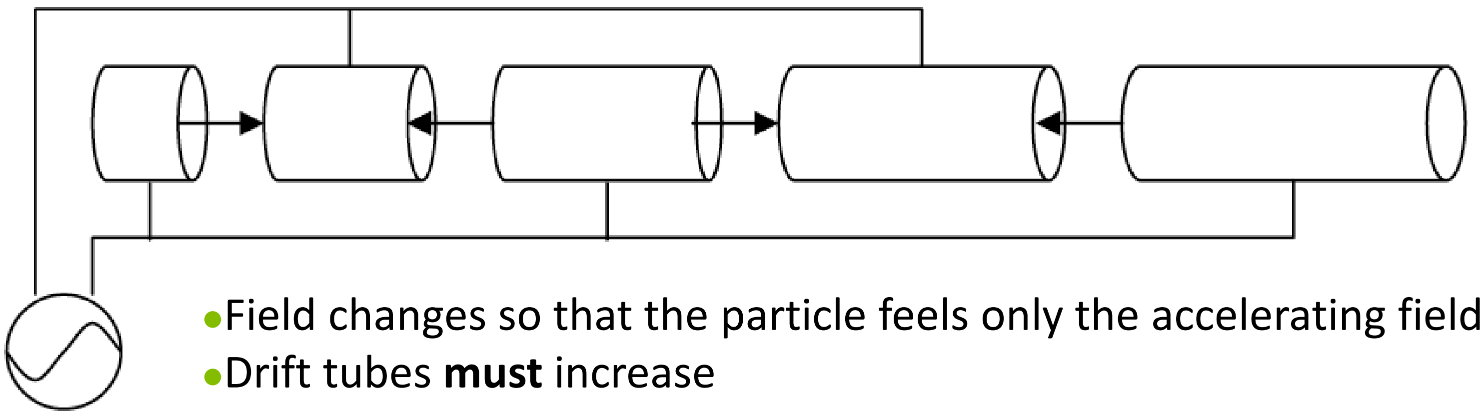
Circular accelerators – multiple passages through the RF cavities

Time varying electric field

- Static systems have voltage limitation
- Oscillating fields overcome this problem
- The acceleration is divided in steps
- One should take into account that the velocity increases during acceleration

Widerøe accelerator

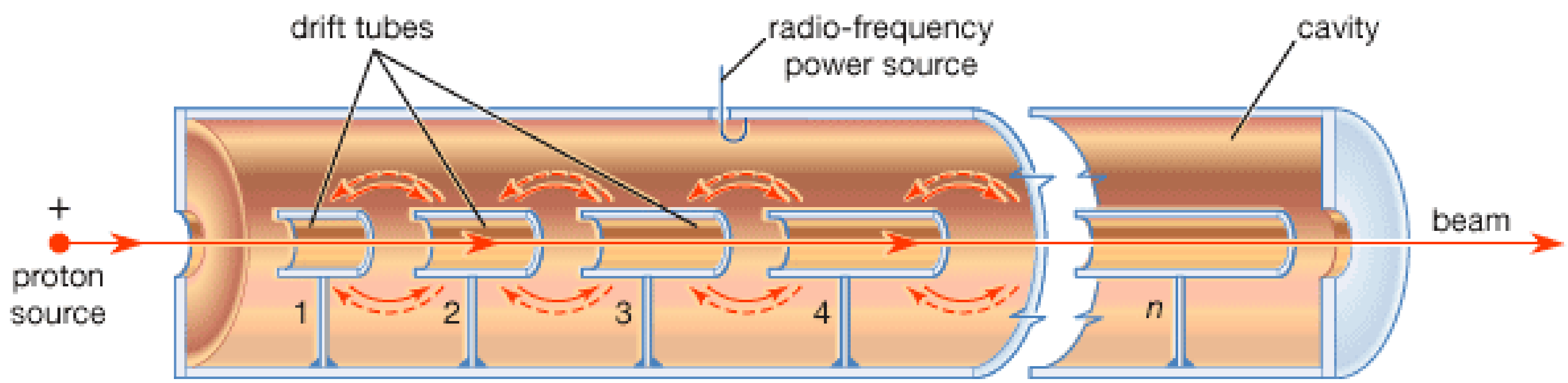
Heavy ions up to 50 keV



- Field changes so that the particle feels only the accelerating field
- Drift tubes **must** increase

At high frequencies the Wideroe accelerator becomes a large emitter of RF power and becomes inefficient.

The Alvarez linac



The accelerator is a **large-diameter tube** within which an electric field oscillates at a high radio frequency.

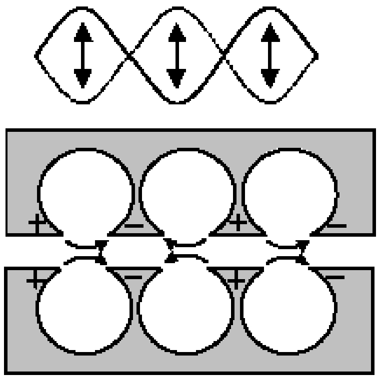
Within the accelerator tube are smaller diameter metallic drift tubes, which are carefully sized and spaced to **shield** the protons from decelerating oscillations of the electric field

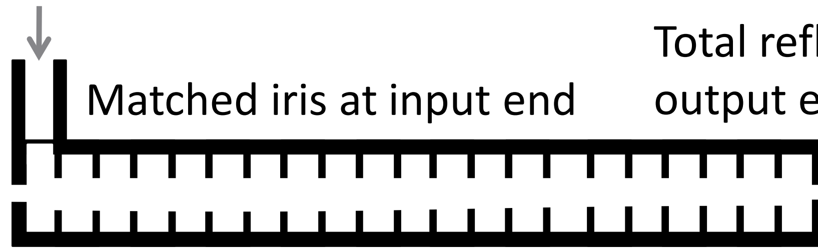
Still used for ions

Different types of linacs

Standing wave

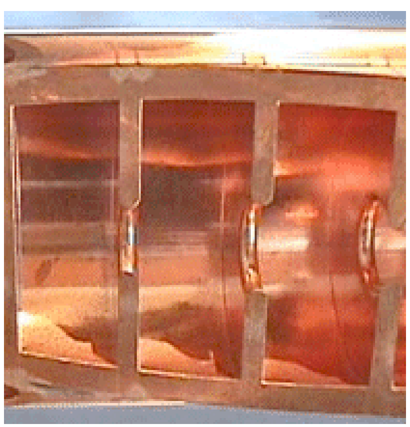
Used for ions and electrons at all energies

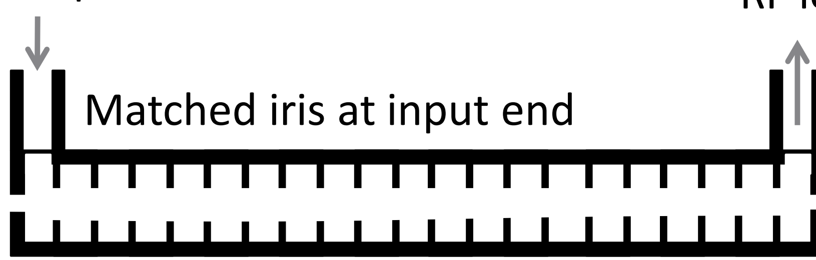




Traveling wave

Used for relativistic electrons



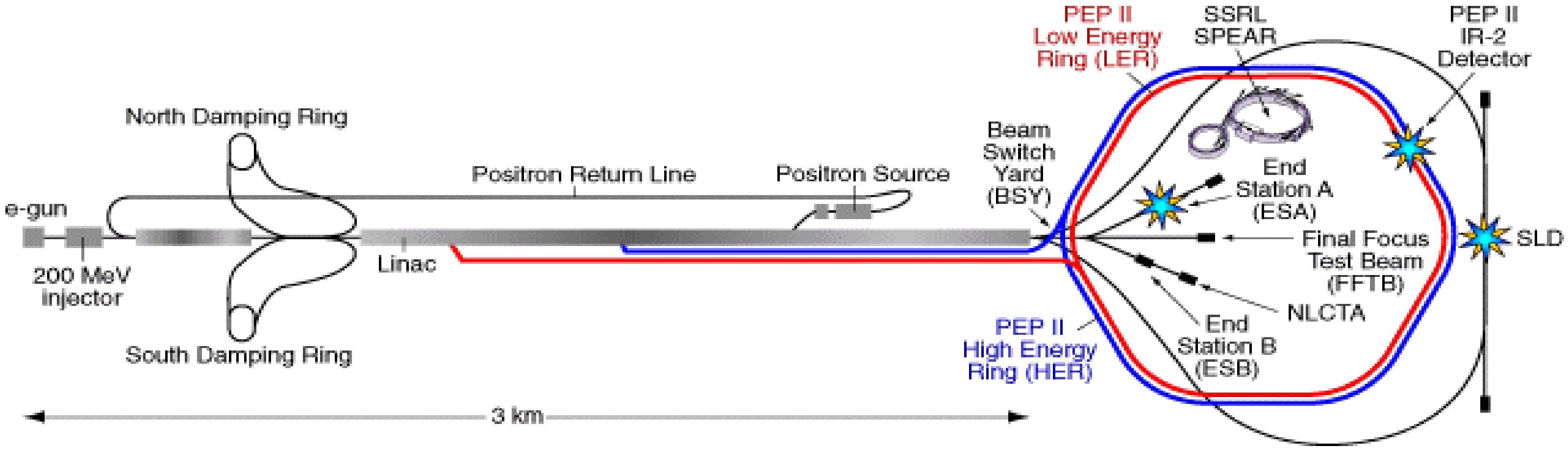


Linac based facilities

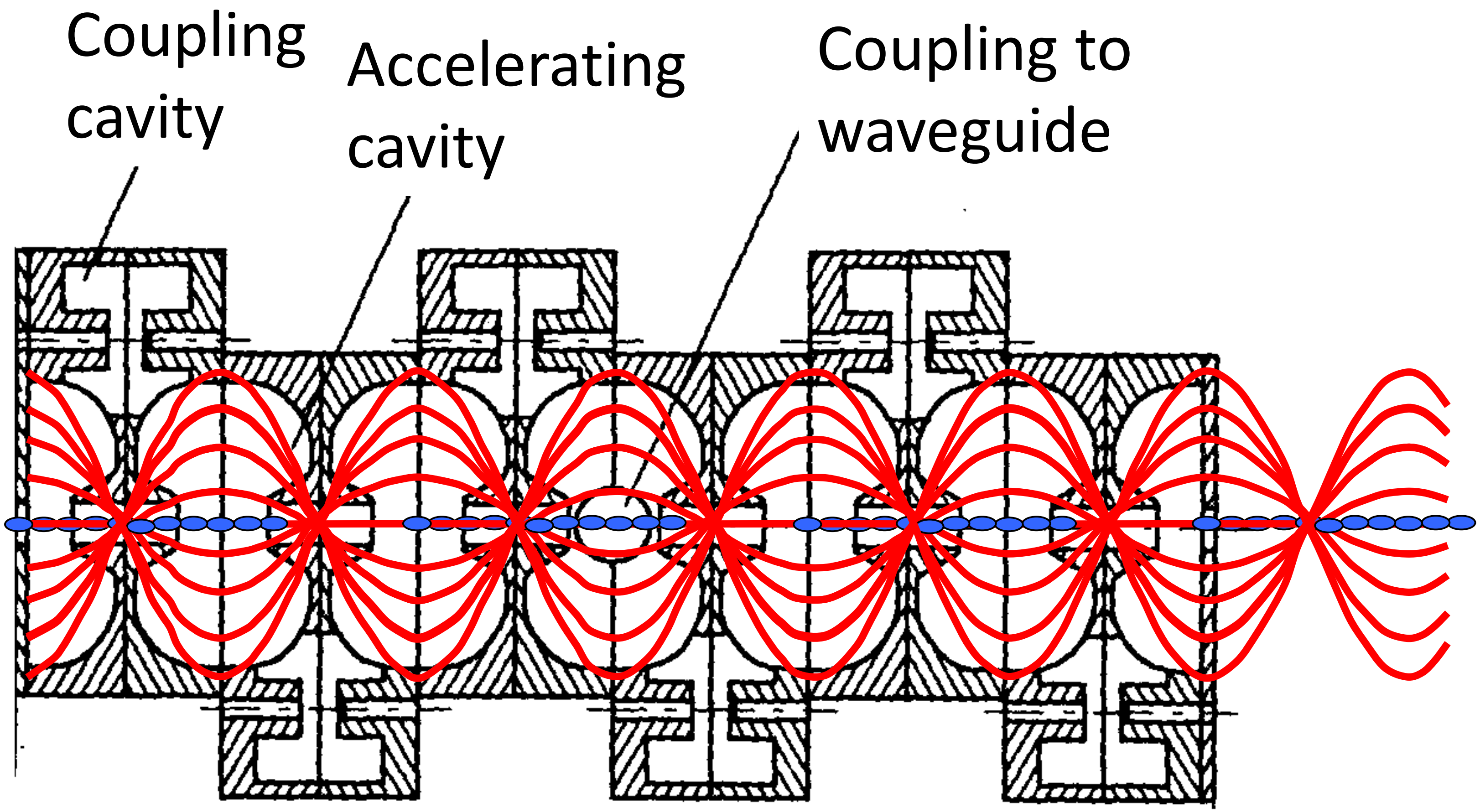
SLAC, San Francisco, California

- Particle physics
- Synchrotron Radiation with LCLS FEL

SLAC National Accelerator Laboratory

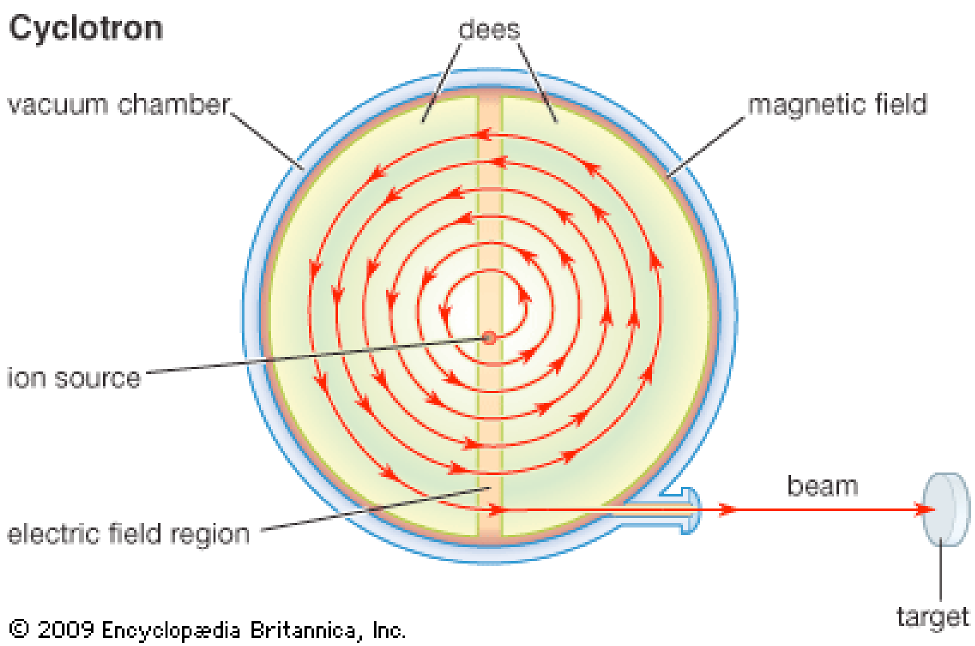


AC acceleration



Circular accelerators

1932: E.O Lawrence patented a cyclotron: "method and apparatus for the acceleration of ions"

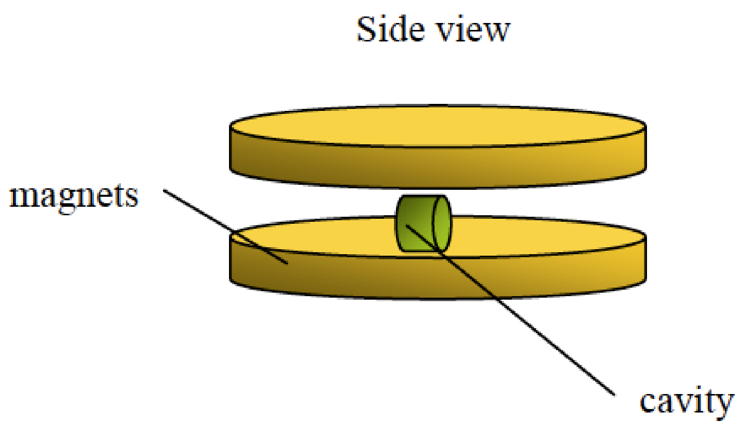


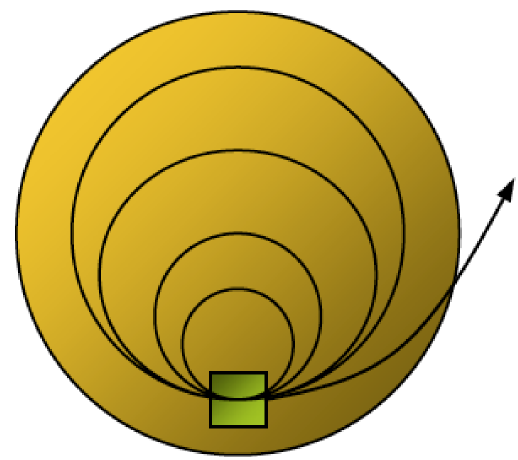
$$T = \frac{2\pi r}{v} = \frac{2\pi m}{qB} = \frac{2\pi m_0 \gamma}{qB}$$

The cyclotron uses Newtonian, or non relativistic, relations for the revolution time. It works for $1 < \gamma < 1.05$.
The peak energy can be increased by having an RF frequency that varies like in the Synchrocyclotron or even better with a magnetic field that is stronger at larger radiuses like in the Isochronous Cyclotron.

- **Possible to reach about 22 MeV**

Microtron





$$T = \frac{2\pi r}{v} = \frac{2\pi m}{qB} = \frac{2\pi W}{qBc^2} = \frac{2\pi}{qBc^2}(W_0 + W_k)$$

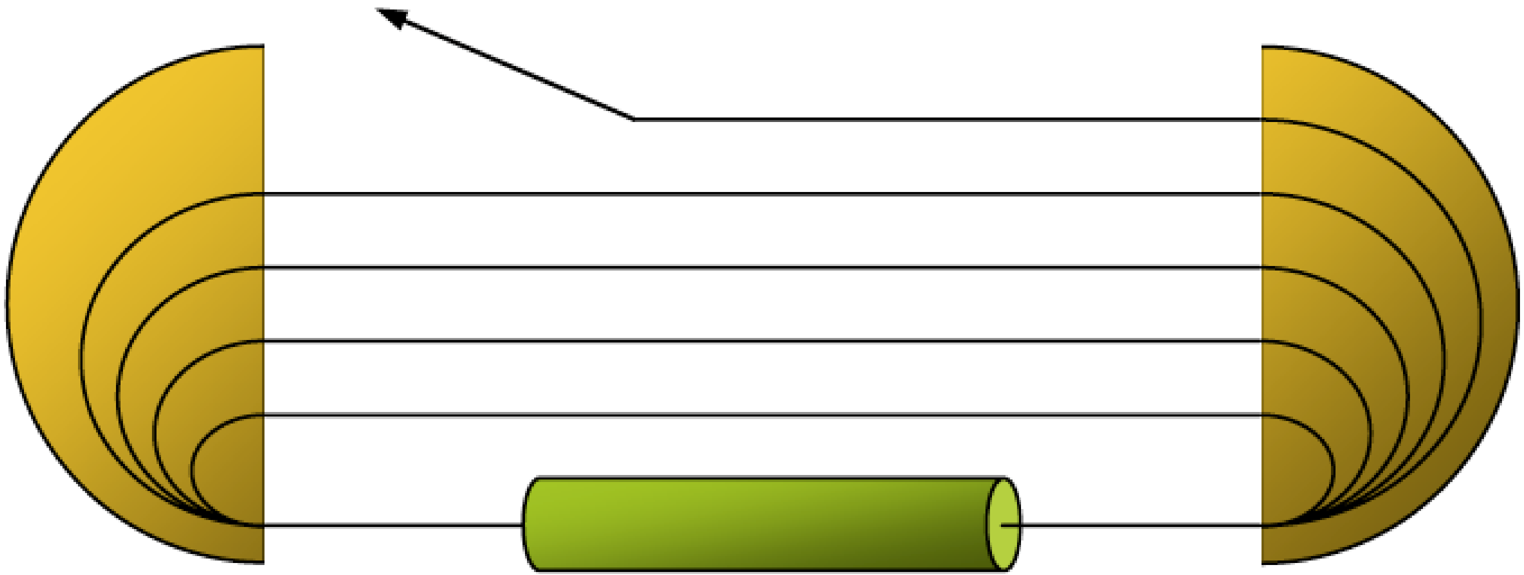
$$\Delta T = \frac{2\pi}{qBc^2}\Delta W$$ Time difference between each revolution

Acceleration when $\Delta T = \frac{k}{f_{RF}}$, k integer

$$n\lambda = n\frac{c}{f} = nc\Delta T = nc\frac{2\pi}{qBc^2}$$

- **Possible to reach about 30 MeV**

Racetrack microtron



Like a microtron but the two halves are split

up to 100MeV

Synchrotron

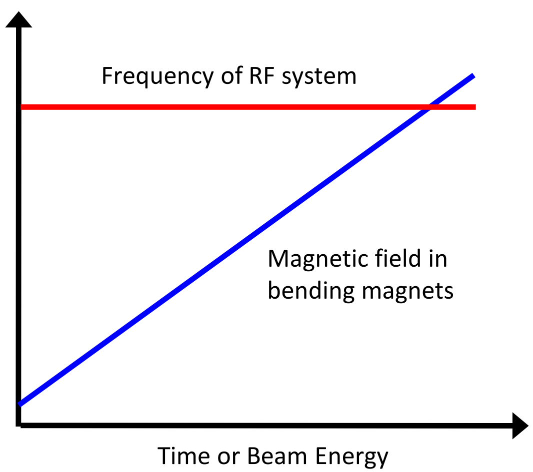
1945: E.M. McMillan and V. Veksler independently developed the concept of synchrotron

The radius is constant while the magnetic field increases

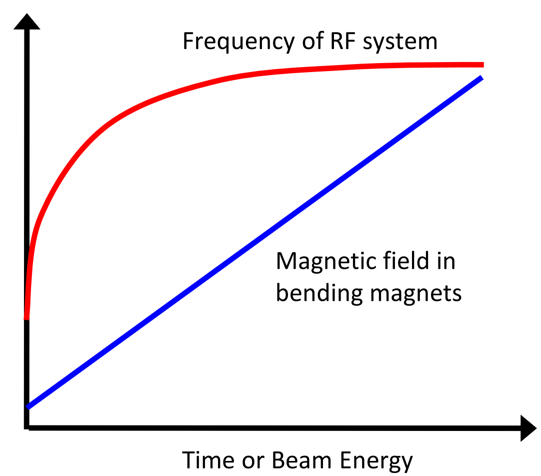
$$\left.\begin{array}{l} \dfrac{dr}{dt} = 0 \\ \dfrac{dB}{dt} \neq 0 \end{array}\right\} \; r = \frac{mv}{qB} = \frac{1}{qB} m_0 \gamma c \sqrt{1 - \frac{1}{\gamma^2}} = \frac{1}{qBc}\sqrt{W^2 - W_0^2}$$

A change in the magnetic field gives a change of energy.

Frequency of RF is **constant** for electron and highly relativistic ion and proton beams.

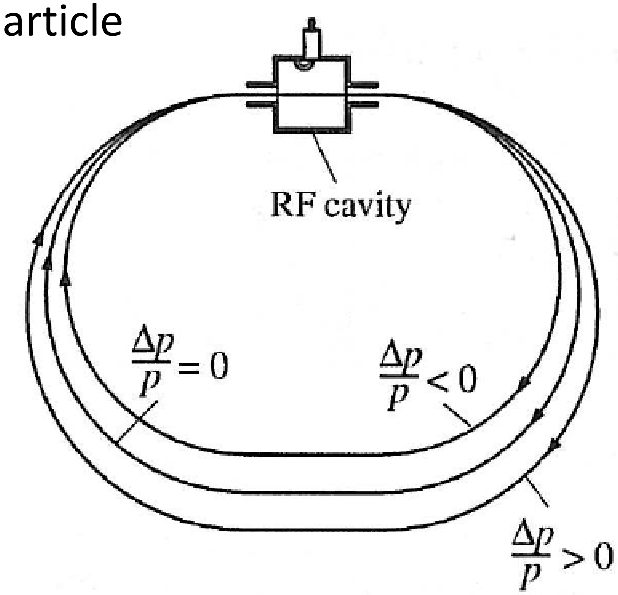


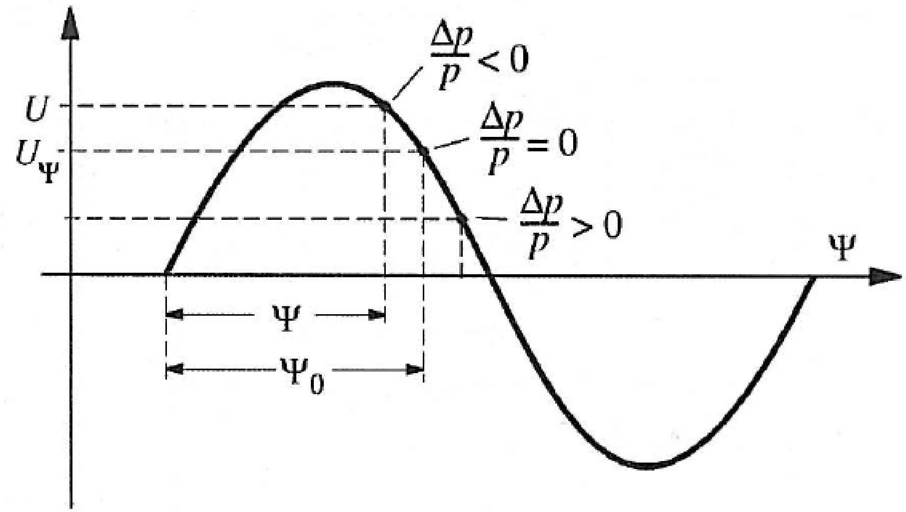
Frequency of RF is **variable** for booster rings for ion and proton beams since *v* ≠ c at start.



Phase stability and synchrotron frequency

The "phase stability" is the capture phenomena occurring around the synchronous particle





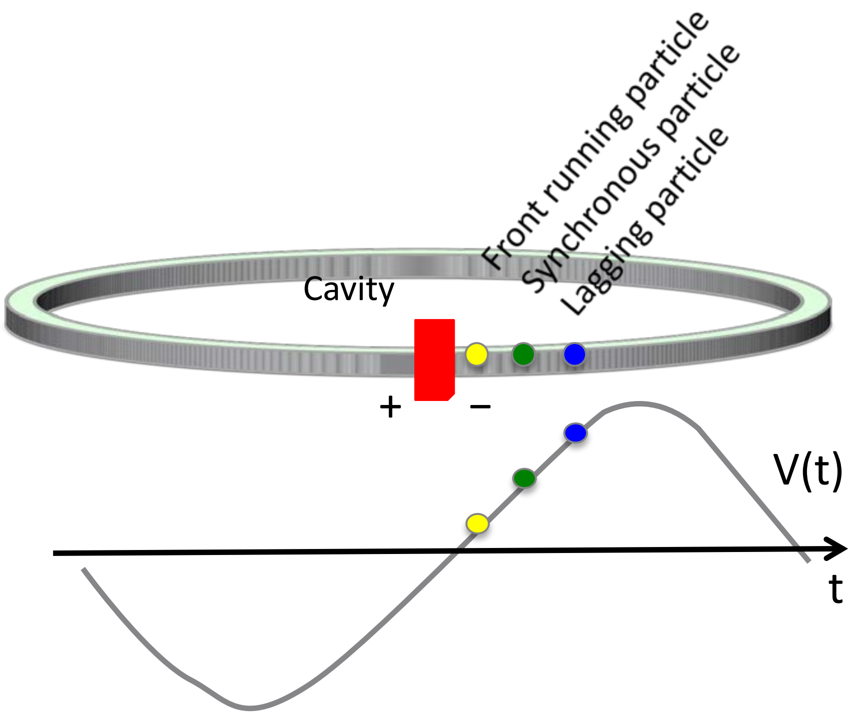
The RF frequency has to be an integer multiple of the revolution frequency

$$\omega_{RF} = h\omega_{rev}$$

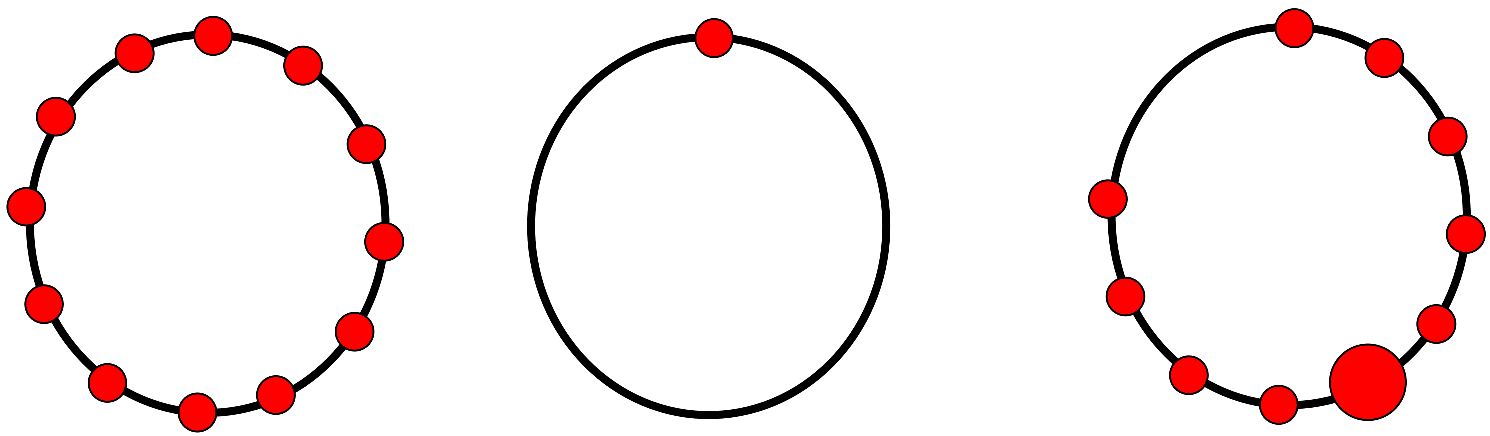
h: harmonic number of the ring

Phase focusing of relativistic particles in a circular accelerator:
The particles oscillate around the **synchronous particle**→Synchrotron oscillations.
The frequency is typically a small fraction of the revolution frequency.

Time structure



The stored beam consists of a series of bunches.
Distance between the bunches = wavelength of RF system
Only a finite number of bunches possible
Every "bucket" does not have to be filled, gaps possible



Emission of radiation and Energy lost per turn

Power radiate by moving charge (Larmor formula) $$P_\gamma = \frac{1}{6\pi\varepsilon_0}\frac{e^2 f^2}{c^3}\gamma^4$$

$f = \frac{v^2}{\rho}$ with $v \approx c$ $\qquad \gamma^2 = \frac{E^2}{c^4 m_0^2}$

r_e classical radius of the electron

$$\boxed{P_\gamma = \frac{2}{3}\frac{r_e c}{(m_0 c^2)^3}\frac{E^4}{\rho^2}}$$

If B is constant, ρ is only a function of momentum

$$\frac{1}{\rho^2} = \frac{B^2 e^2}{p^2} = \frac{B^2 e^2 c^2}{(pc)^2} \approx \frac{B^2 e^2 c^2}{E^2}$$

$$\boxed{P_\gamma = \frac{2}{3}\frac{r_e e^2}{(m_0 c)^3}E^2 B^2}$$

In order to provide the energy lost (i.e. the voltage required to keep the beam stored), one needs to calculate what is the energy radiated by a particle on each turn

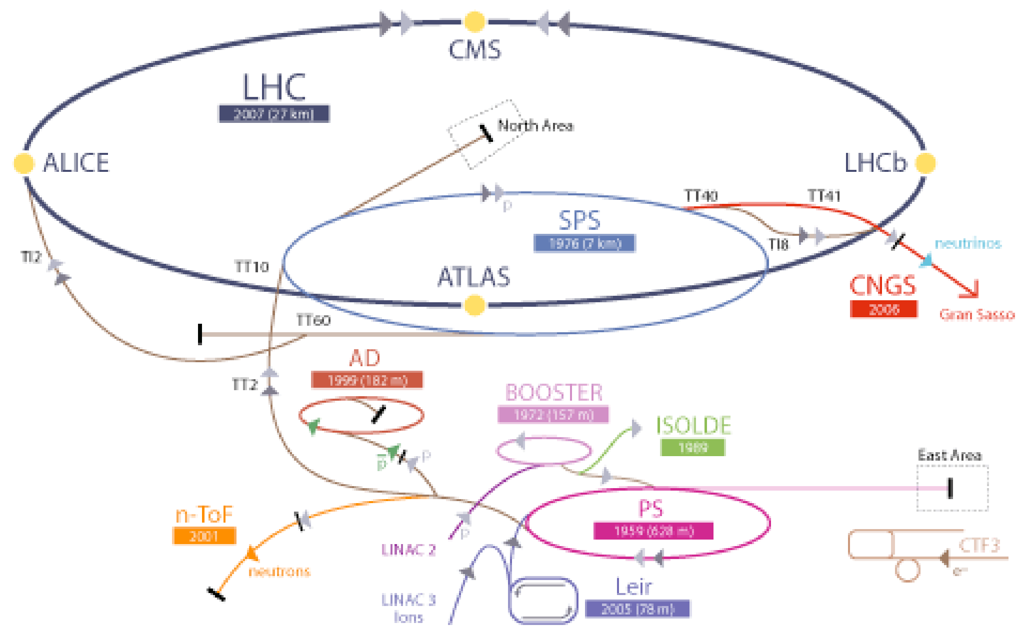
Energy = power (P) × revolution time ($2\pi R/\beta c$) $$U_0 = \frac{4\pi}{3}\frac{r_e}{(m_0 c^2)^3}\frac{E^4}{\rho^2}$$

For electron machines above 100 GeV is not practical to scale energy and the radius linearly with the energy

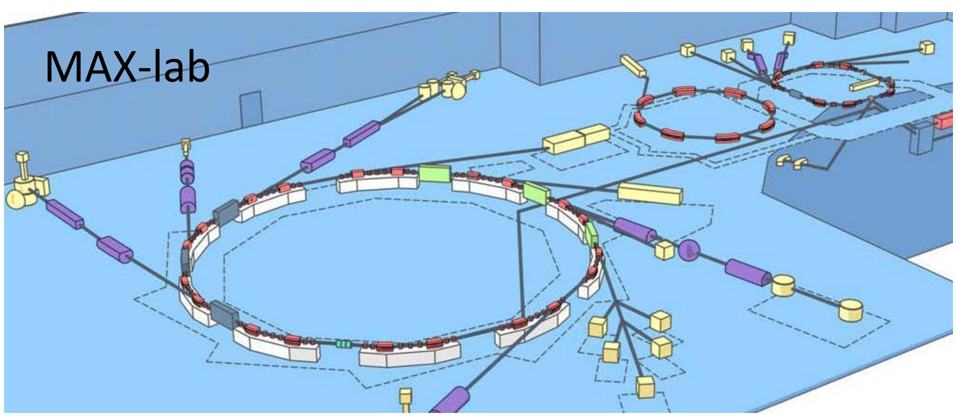
Synchrotrons

CERN Accelerator Complex

Large collider accelerators

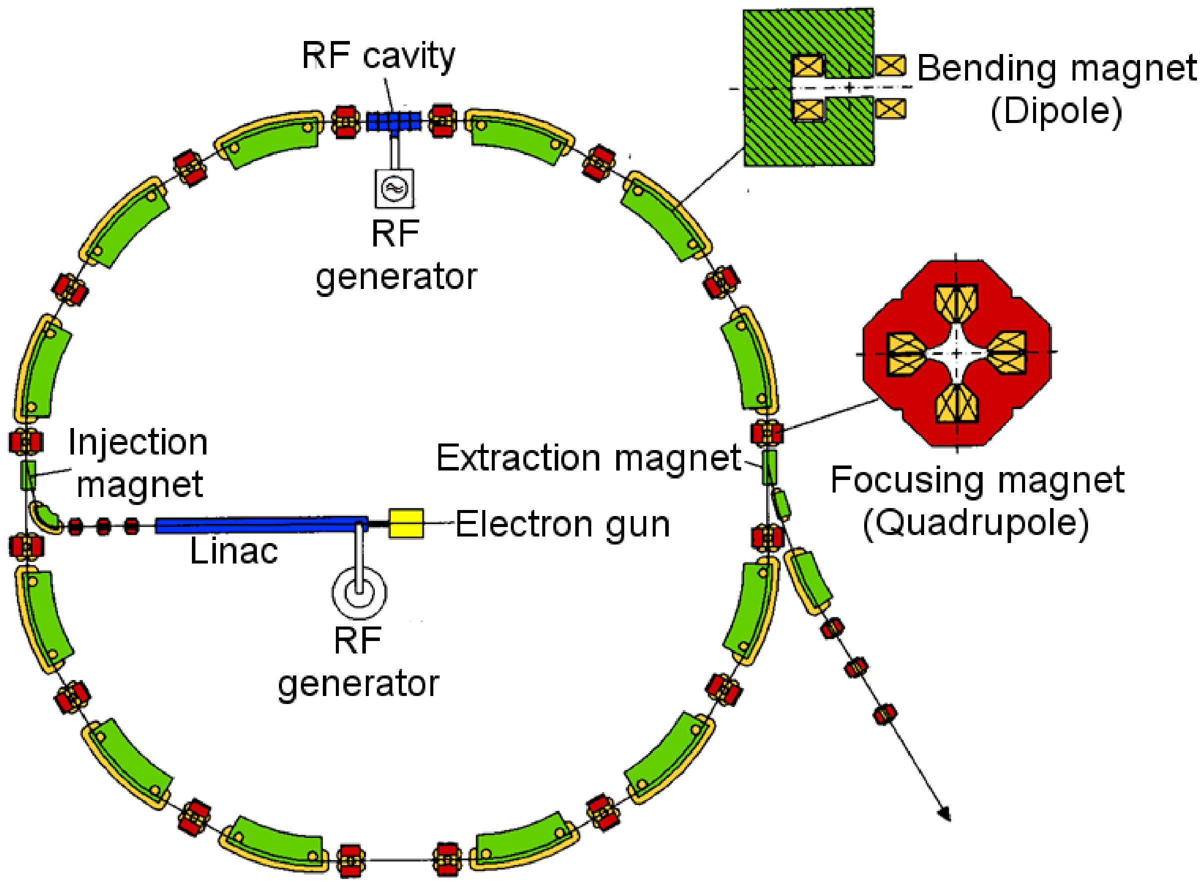


Storage rings for Synchrotron Radiation production

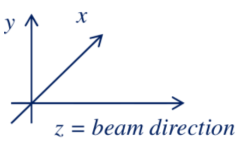


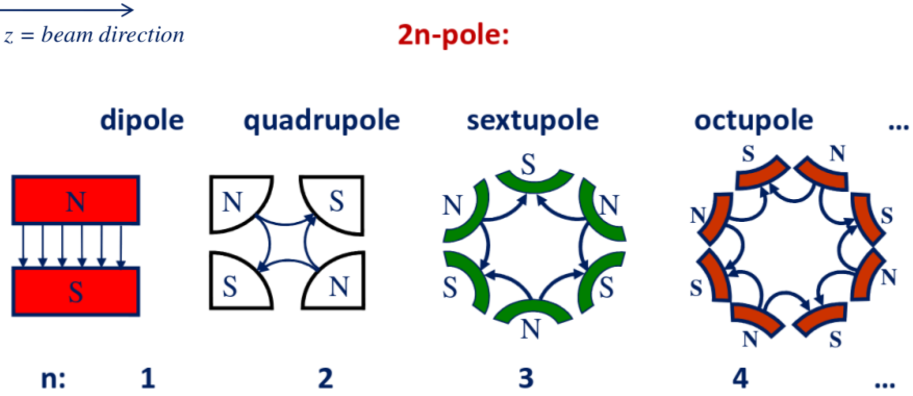
MAX IV

How does it look like a synchrotron?



Types of magnets





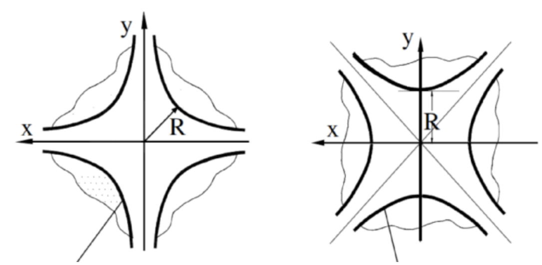
USPAS14, Fundamental Acc. Physics and Technology

- Electromagnets
- Permanent magnets

$$B_y(x) = \boxed{B_{y0} + \frac{dB_y}{dx}x} + \boxed{\frac{1}{2!}\frac{d^2B_y}{dx^2}x^2 + \frac{1}{3!}\frac{d^3B_y}{dx^3}x^3} + \ldots$$

Linear optics (steering):
→ dipoles
→ quadrupoles

Higher order optics (compensation or errors):
→ sextupoles
→ octupoles



- Normal: gap in hor. plane
- Skew: rotate around beam axis by π/2n angle

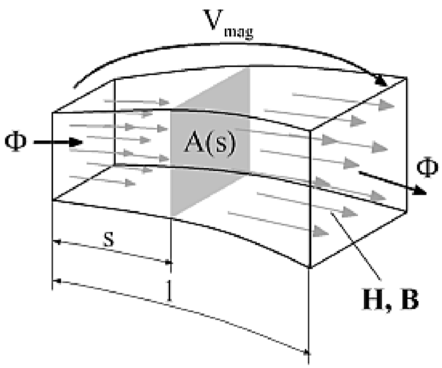
Recapitulation

$\mu_0 = 4\pi * 10^{-7}$ vacuum permeability

The permeability:	$\mu = \mu_r/\mu_0$	Vs/Am
Magnetic flux:	Φ	Wb = Vs
The magnetic flux density:	B	$T = Vs/m^2$
The magnetic fields strength:	H	A/m

vacuum $\mu_r = 1$
iron $\mu_r = 2000$

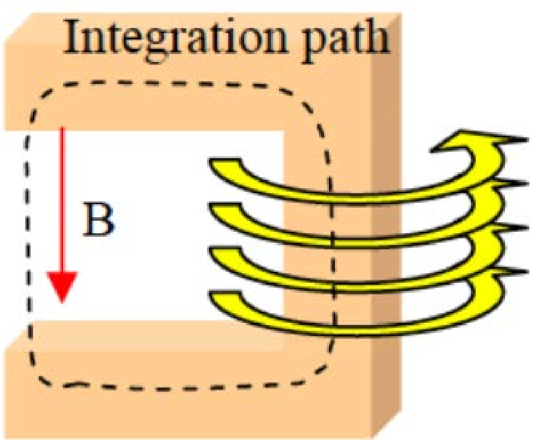
Magnetic flux:



Ampère's circuital law:

$$\oint \vec{H} d\vec{s} = \int \vec{j} d\vec{A} = nI$$

n – number of coil windings

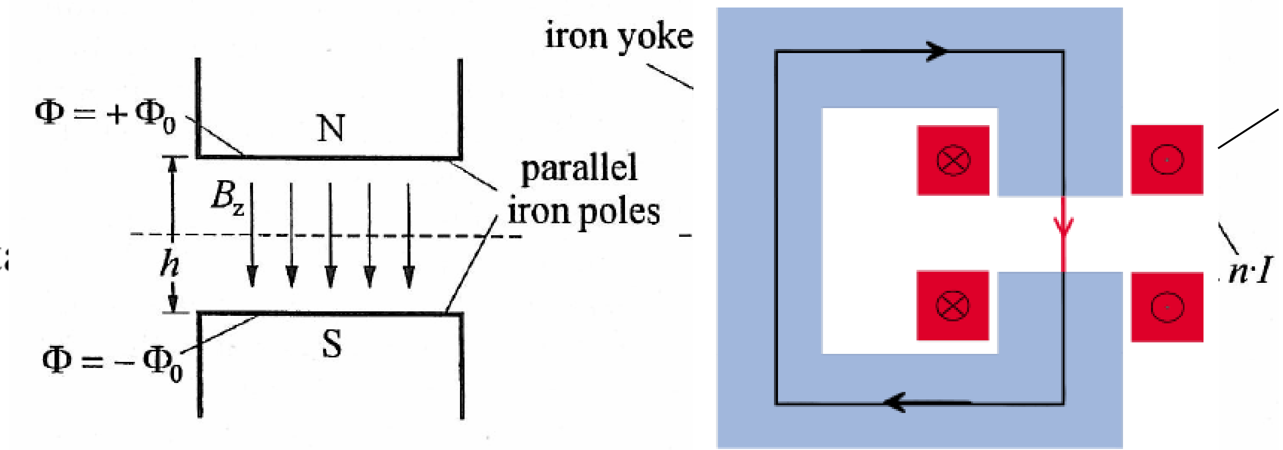


$B = \mu H$

Dipole magnet field

$\mu_{Fe} \approx 2000\ \mu_{air}$

$$B_x = 0$$
$$B_y = B_0 = \text{const}$$



K.Wille 'The physics of Particle Accelerators'

Ampère's circuital law: $\oint H ds = hH_{gap} + lH_{Fe} = nI$

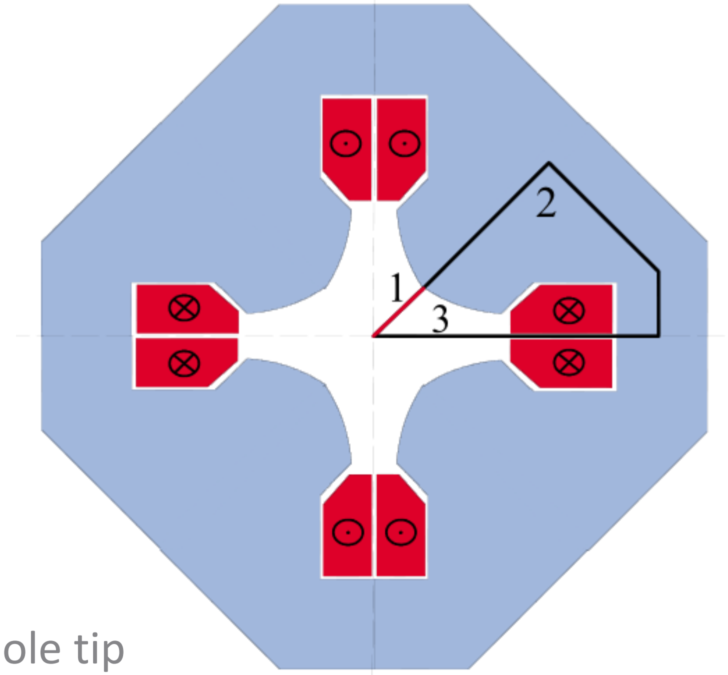
The magnetic flux density (B) at the two sides of the iron-air interface is constant: $H_{gap}\dfrac{\mu_{air}}{\mu_0} = H_{Fe}\dfrac{\mu_{Fe}}{\mu_0}$

$$\oint H ds \approx hH_{gap} = h\frac{B}{\mu_0} = nI \Rightarrow \boxed{B = \frac{nI\mu_0}{h}}$$

Quadrupole magnet field

Ampère's circuital law:

$$\oint \vec{H} d\vec{s} = \int_1 \vec{H}_1 d\vec{s} + \underbrace{\int_2 \vec{H}_2 d\vec{s}}_{\text{small}} + \underbrace{\int_3 \vec{H}_3 d\vec{s}}_{0} = nI$$



$$B_x = G\,y \qquad B_y = G\,x \qquad \mathrm{G} = \text{constant}$$

$$B_r = \sqrt{B_x^2 + B_y^2} = Gr$$

In the direction of a pole tip

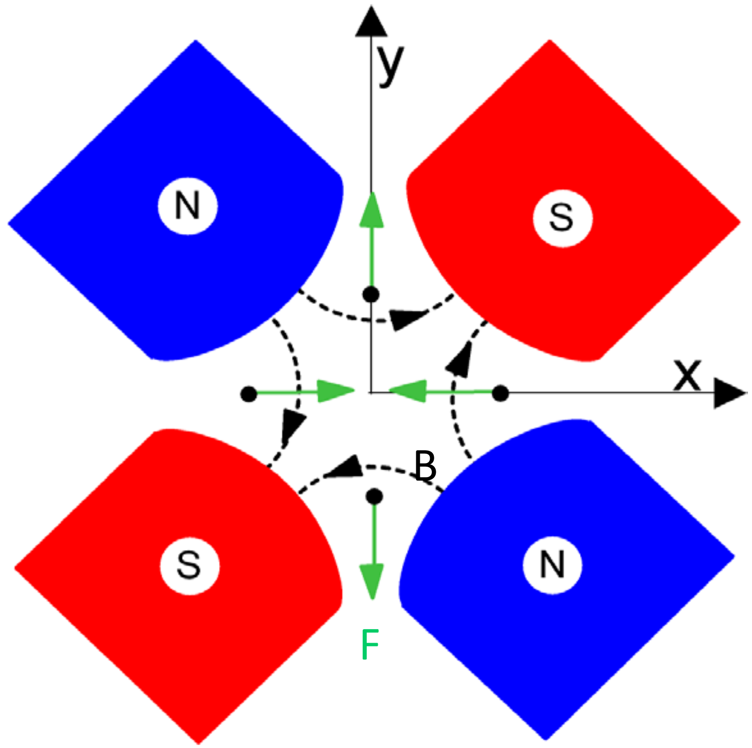
$$nI = \int_1 H_1 ds = \frac{G}{\mu_0}\int r dr = \frac{G r_0^2}{2\mu_0}$$

S. Russenschuck, DESIGN OF ACCELERATOR MAGNETS

$$G = \frac{2nI\mu_0}{r_0^2}$$

Field gradient

Quadrupole focusing

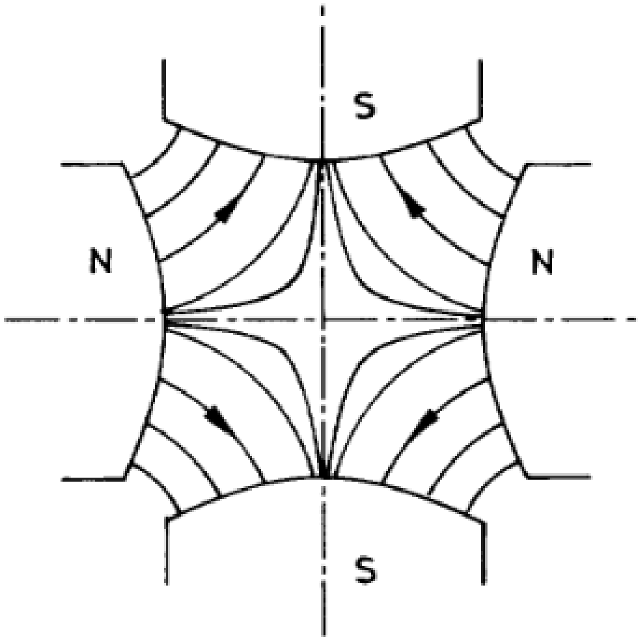


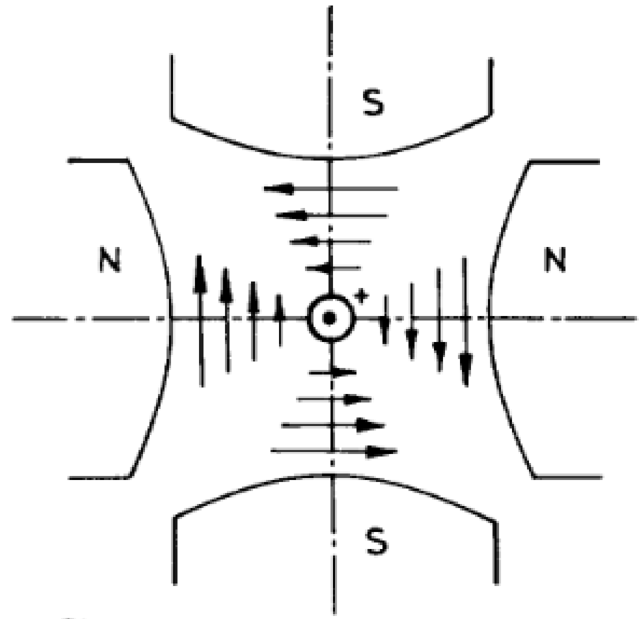
$$B_y \propto x \text{ and } B_x \propto y$$

$$F_x \propto -x \text{ and } F_y \propto y$$

A quadrupole magnet will focus in one plane and defocus in the other!

Skew quarupoles

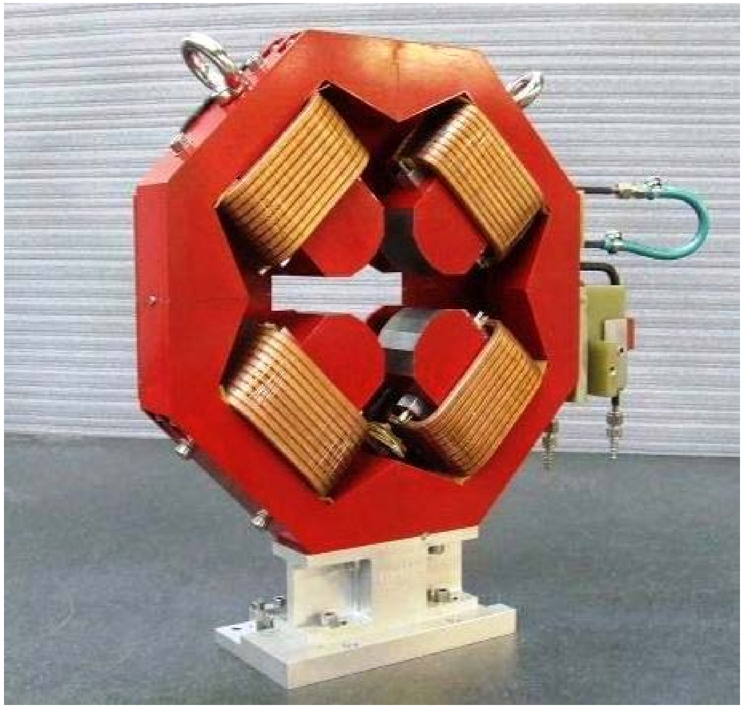




E.Wilson, LINEAR COUPLING

Introduces the coupling of horizontal and vertical motion

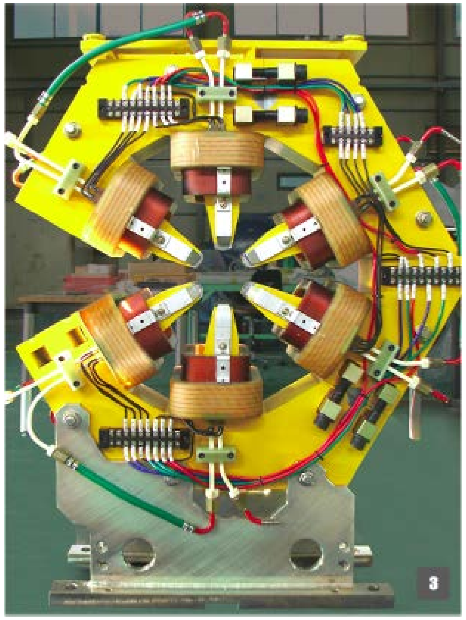
How they look like in real life



[http://www.stfc.ac.uk]

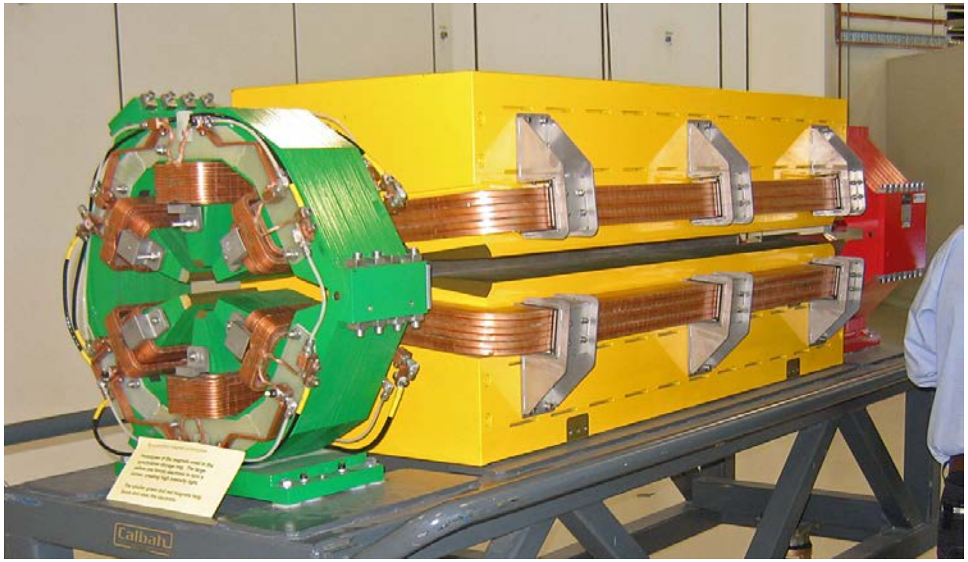
Quadrupole

Sextupole



K.R TECH

Dipole and sextupole



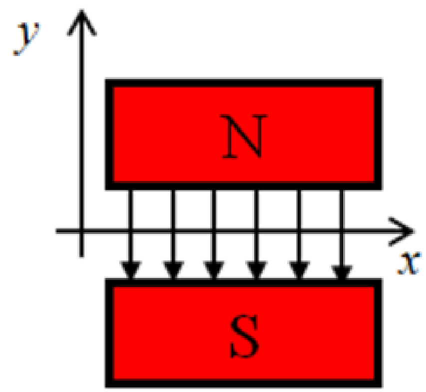
MAX III magnet blocks

Same technology is used in MAX IV

Properties

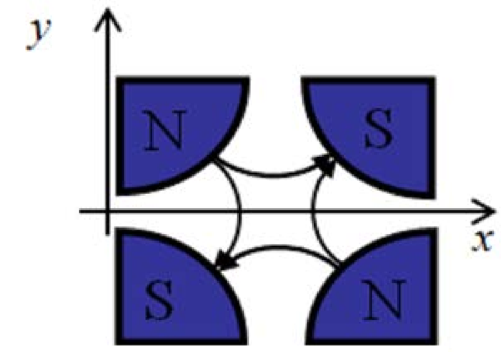
Dipoles : steering the beam

$$B_x = 0$$
$$B_y = B_0 = \text{constant}$$



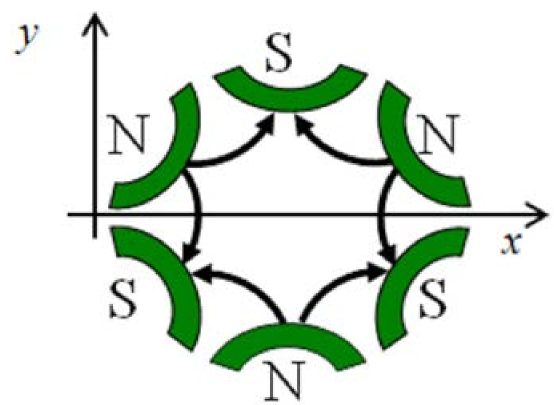
Quadrupoles: focusing

$$B_x = G\,y \qquad \mathrm{G} = \text{constant}$$
$$B_y = G\,x$$



Sextupole: chromatic correction and control of nonlinear dynamics

$$B_x = 2S\,x\,y \qquad \mathrm{S} = \text{constant}$$
$$B_y = S\left(x^2 - y^2\right)$$

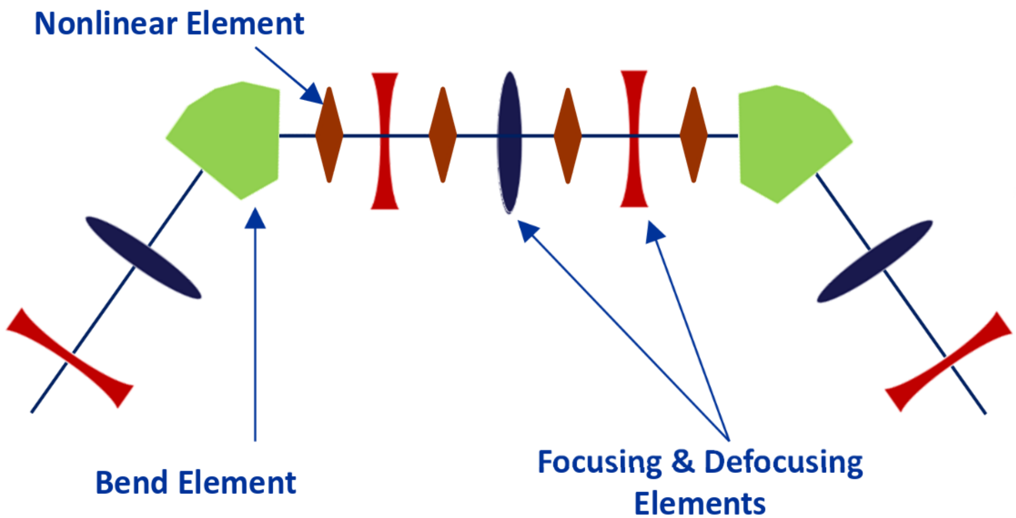


Particle steering tools

The particles should move on a **ideal orbit**

The magnets bend the trajectory and focus the particles

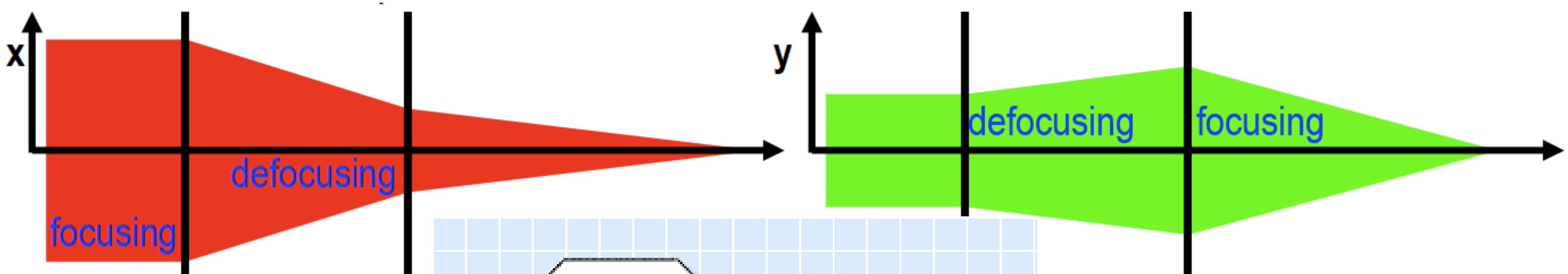
The **lattice** is the arrangement of magnets that guide and focus the beam
→beam optics (tomorrow)



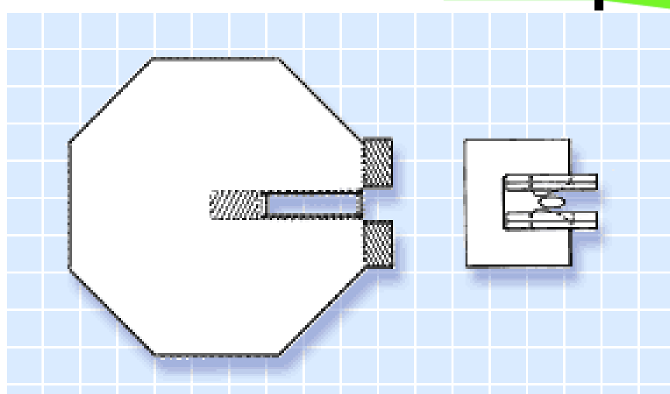
Strong focusing

1952: Courant, Livingston, and Snyder: theory of strong focusing with **discrete quadrupole magnets** for the <u>focusing</u> and **dipole magnets** for the <u>bending</u>.

Two successive elements, one focusing the other defocusing, can focus in both planes:



Weak focusing
Beam pipe height: 15cm



Strong focusing
Beam pipe height: 4 cm

Today: only strong focusing is used

G. Hoffstaetter, Class Phys 488/688 Cornell University

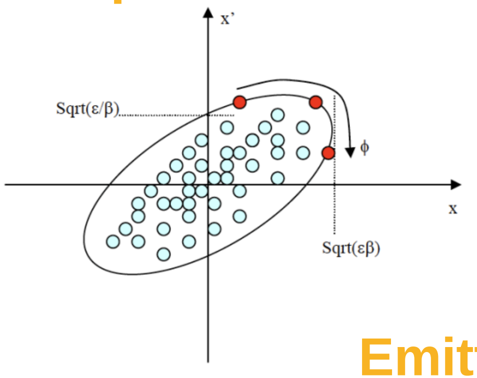
Appetizer

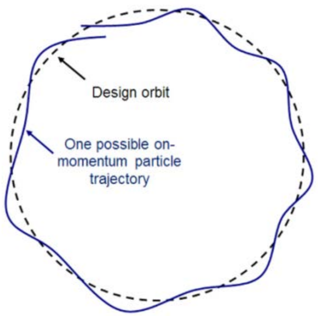
Matrix notation

$$\begin{pmatrix} u(s) \\ u'(s) \end{pmatrix} = \begin{pmatrix} C(s) & S(s) \\ C'(s) & S'(s) \end{pmatrix} \begin{pmatrix} u(0) \\ u'(0) \end{pmatrix}$$

Hill's equations of linear particle motion

Phase space





Betatron oscillations

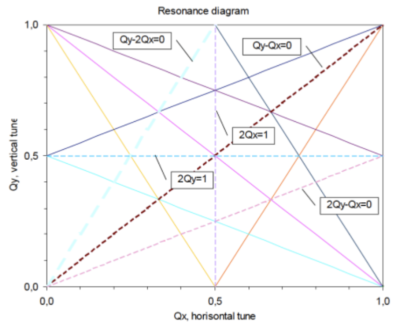
$$u(s) = \sqrt{\varepsilon_u \beta_u(s)} \cos[\varphi_u(s) - \varphi_u(0)]$$

$$u'(s) = -\sqrt{\frac{\varepsilon_u}{\beta_u(s)}} \{\alpha_u(s)\cos[\varphi_u(s) - \varphi_u(0)] + \sin[\varphi_u(s) - \varphi_u(0)]\}$$

$$u = x, y$$

Emittance

Tune



Dispersion

Chromaticity

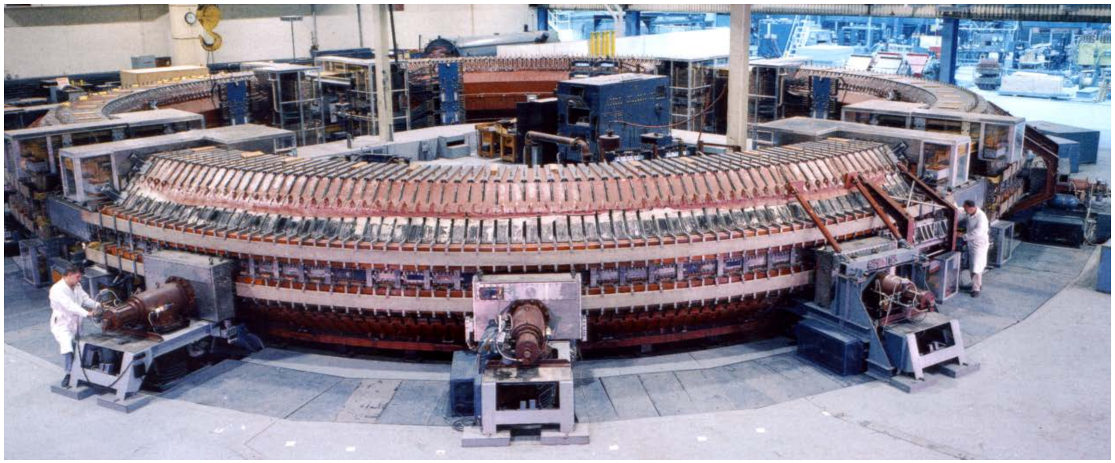
Momentum compaction

Acknowledgements

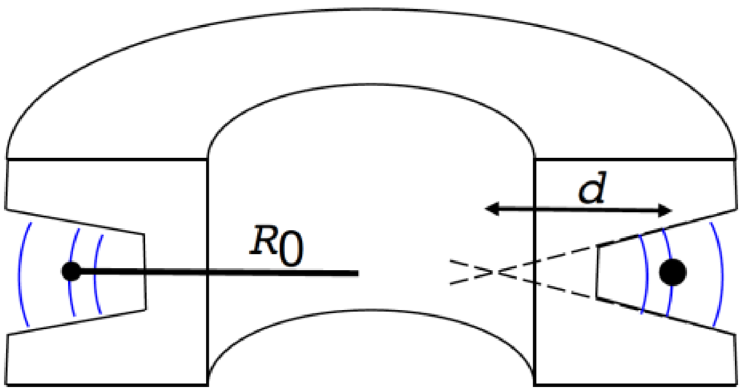
The material used for this lecture comes from E. Wallén, S. Werin and Galina Skripka

Backup

Weak focusing



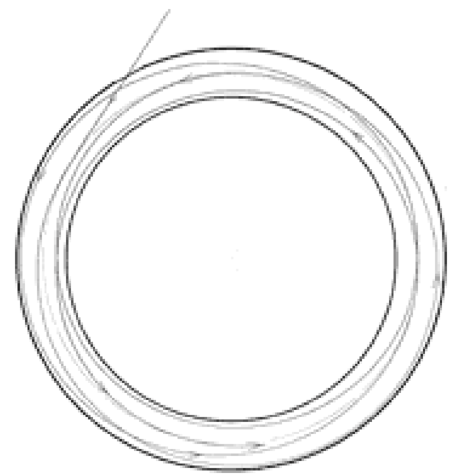
The Cosmotron: 3.3 GeV proton synchrotron at Brookhaven, New York (1952)
Weight: 4000 tons
Magnet aperture: 20 by 60 cm, internal beam pipe height: 15cm



$$n \approx \frac{R_0}{d} \quad \boxed{\text{must have } 0 \le n \le 1 \text{ for stability}}$$

"Minuses":

- Large beam
- Large vacuum chamber
- Large magnet aperture



Weak focusing accelerator