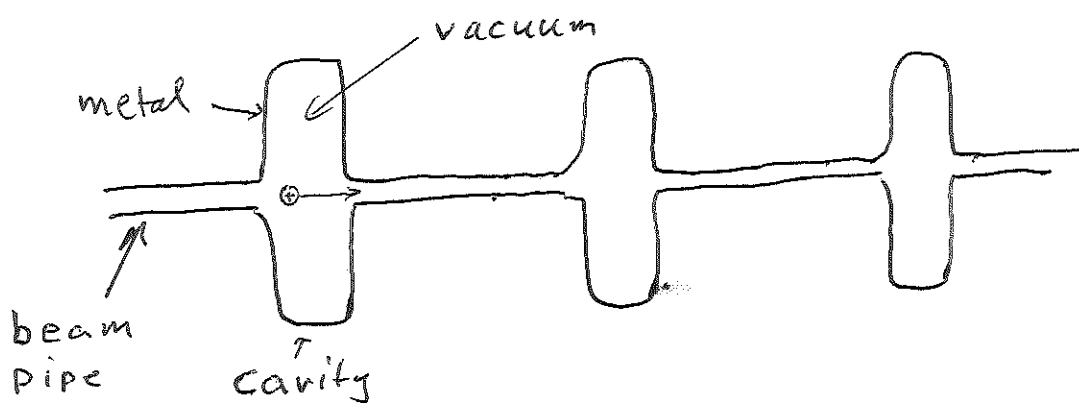


Particles are accelerated by electric forces!

$$\oplus \xrightarrow{\vec{F} = q\vec{E}} \\ q$$

Electrostatic accelerators can only accelerate up to 30 MeV. For larger energies we need electromagnetic waves!

Ex Let's follow a proton in a linear accelerator



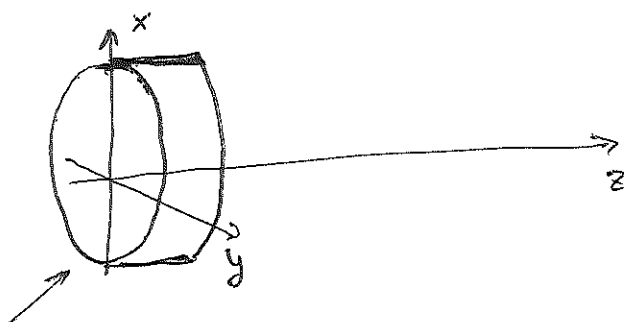
No forces in the beam pipe \Rightarrow constant speed
Electric field in cavity \Rightarrow acceleration

Cavities

A cavity has resonance frequencies. f_1, f_2, f_3, \dots
To each resonance there is an electromagnetic standing wave. We call such a wave a mode,

We use one of the modes for acceleration of the particles.

Example The pill-box cavity



Pill-box = circular cylinder

The perfect mode for acceleration is the fundamental mode. It is called TM_{010} .

The electric field of this mode is given by

$$\vec{E}(\rho, t) = E_0 J_0(k\rho) \cos \omega t \cdot \hat{z}$$

where $\rho = \sqrt{x^2 + y^2}$

$k = \frac{\omega}{c} = \text{wavenumber}$

$\omega = 2\pi \cdot f = \text{angular frequency}$

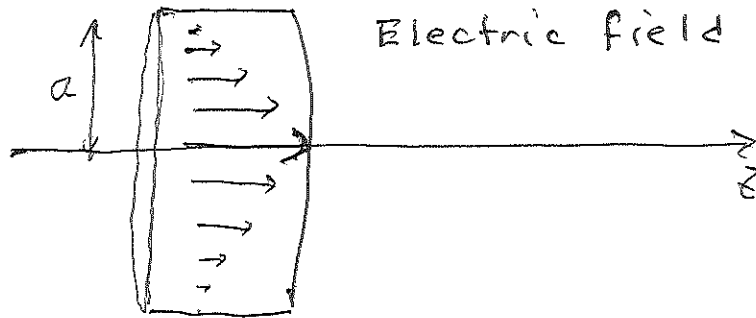
$c = \text{speed of light}$

$J_0(k\rho) = \text{Bessel function of order zero.}$

$\hat{z} = \text{unit vector}$

$E_0 = \text{amplitude}$

$$E = 0$$

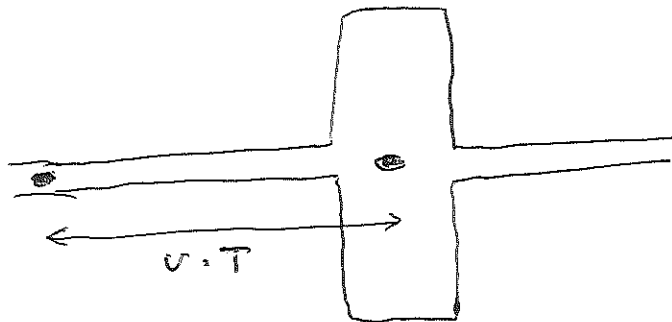


The frequency is $f = \frac{c}{2\pi} \cdot \frac{2.405}{a}$

$$a = 10 \text{ cm} \Rightarrow f = 1.15 \text{ GHz}$$

Bunches

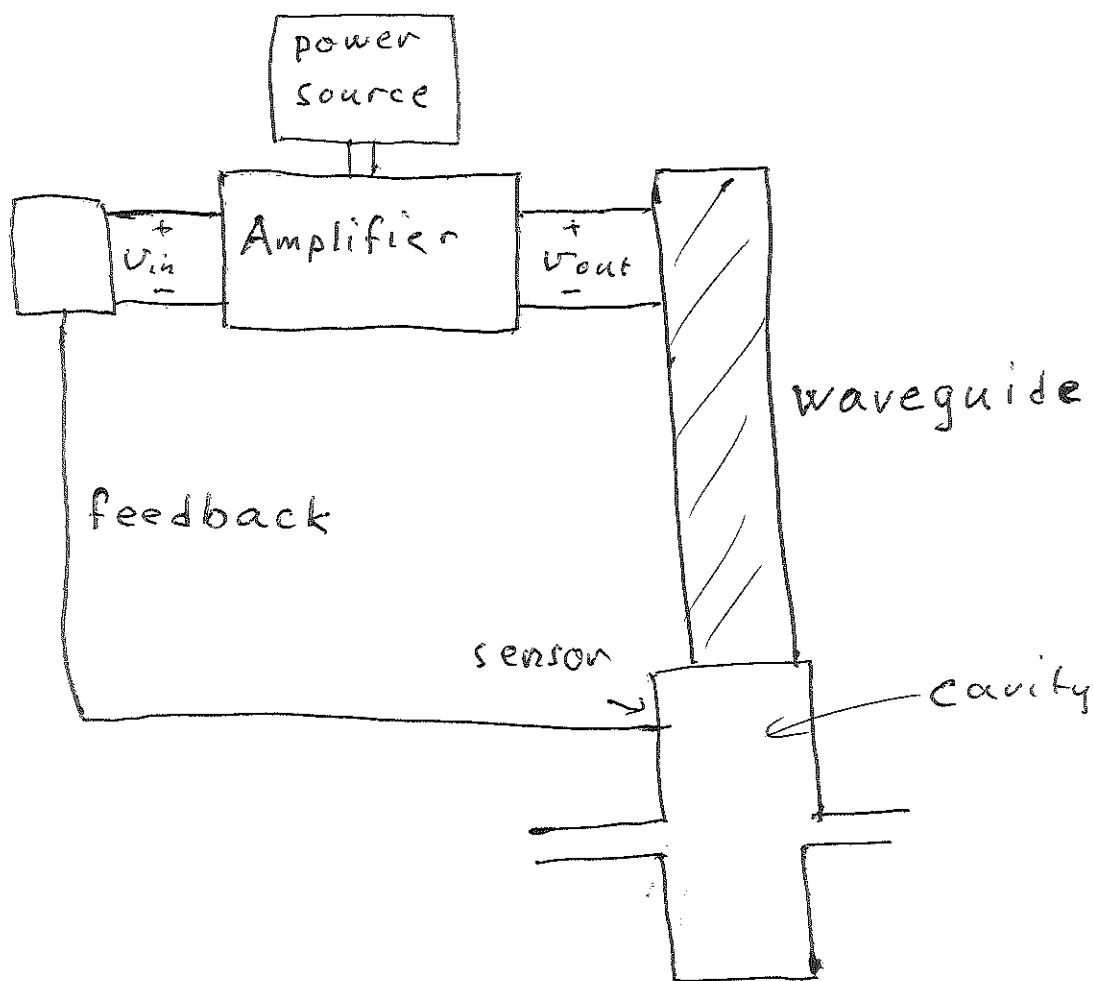
The beam of particles must be bunched
 Time between bunches = $T = \frac{1}{f}$



RF-system (Radio frequency system)

The RF-system generates the waves and deliver them to the cavity.

Outline of Rf-system



$$V_{in}(t) = V_0 \cos(\omega t + \varphi) = \text{input voltage}$$

V_0 = amplitude

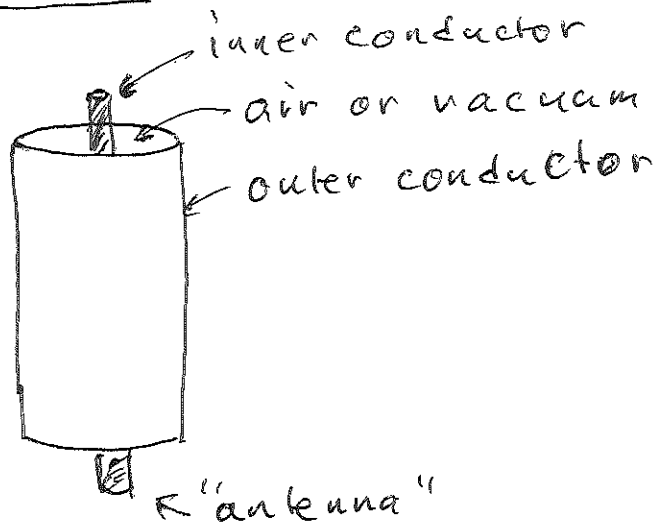
φ = phase angle

$$V_{out}(t) = V_1 \cos(\omega t + \varphi) = \text{output voltage}$$

$$\text{gain} = \frac{V_1}{V_0}$$

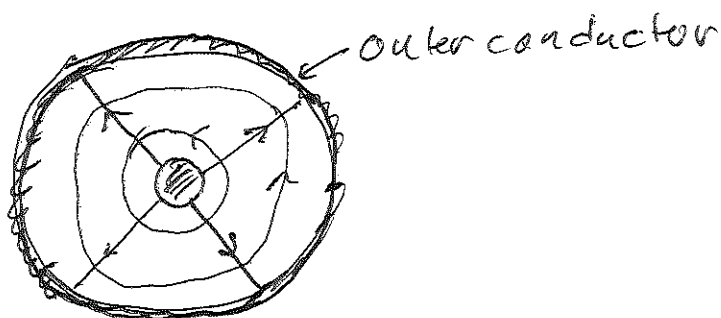
Waveguides

Coaxial



- + All frequencies can propagate
- + All waves propagate with the same speed
- Quite large losses (waves are attenuated)

Ex



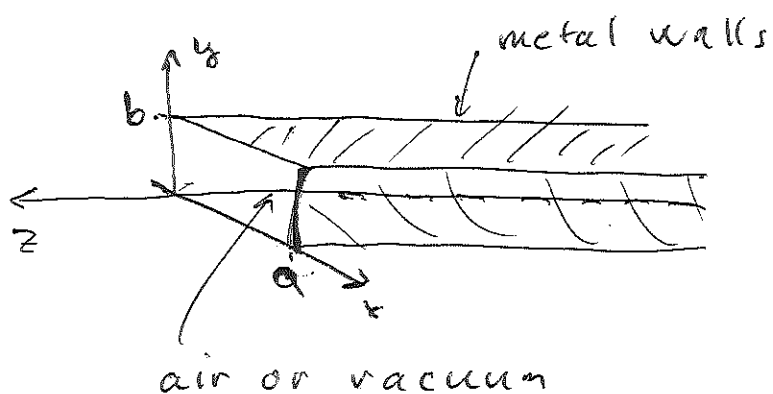
$$\vec{E}(\rho, z, t) = E_0 \frac{a}{\rho} \cos(\omega t - kz) \hat{\rho}$$

$$\vec{H}(\rho, z, t) = H_0 \frac{a}{\rho} \cos(\omega t - kz) \hat{\phi}$$

$$E_0 = \eta_0 H_0$$

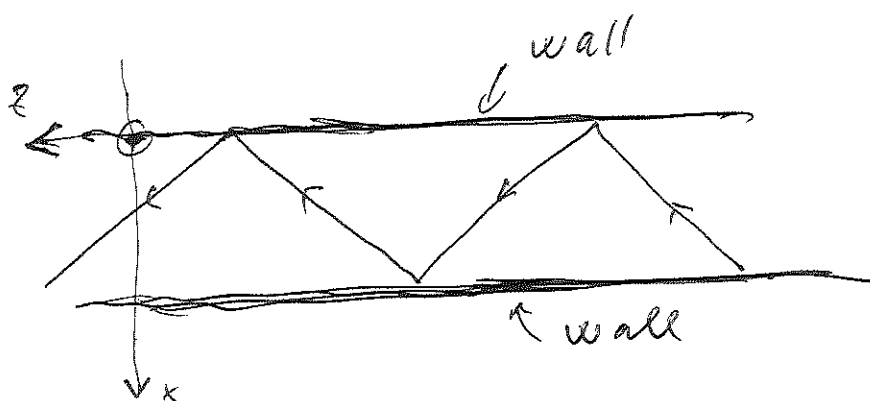
$\eta_0 = 120 \pi =$ wave impedance of vacuum
 $a =$ radius of inner conductor

Rectangular



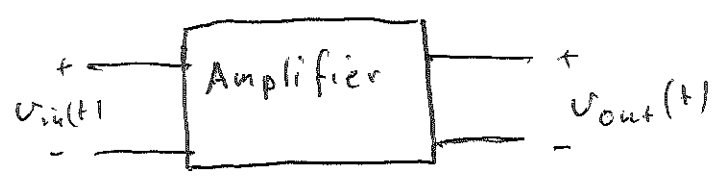
+ low losses

- Only waves with frequencies $> \frac{c}{2a}$ can propagate
- Phase and group speeds are frequency dependent \Rightarrow dispersion



waves bounce between walls!

The amplifier



$$v_{in}(t) = V_0 \cos(\omega t + \phi)$$

$$v_{out}(t) = V_1 \cos(\omega t + \phi)$$

$$\text{Gain} = \frac{V_1}{V_0}$$

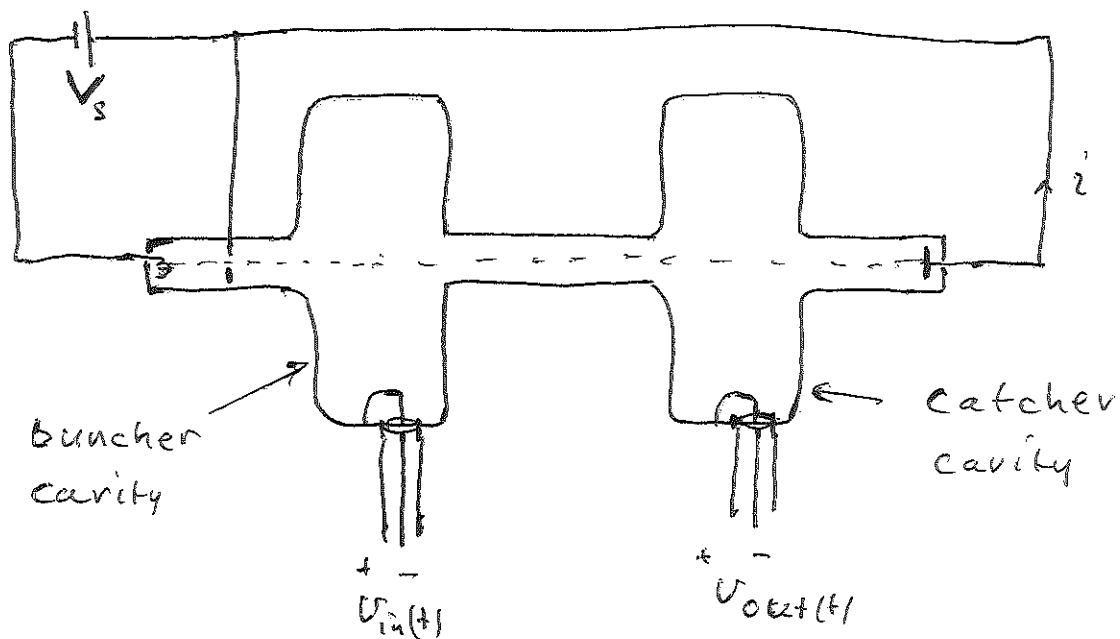
$$\text{Gain}_{dB} = 20 \log\left(\frac{V_1}{V_0}\right)$$

$$\Leftrightarrow \text{Gain}_{dB} = 10 \log\left(\frac{P_{out}}{P_{in}}\right)$$

P = power

Ex Low gain \Rightarrow use solid state
 Large gain \Rightarrow large $P_{out} \Rightarrow$ use klystron!

The klystron



- Heat filament \Rightarrow electrons that accelerate to the right. Electric energy \rightarrow kinetic energy
- Add $v_{in}(t) \Rightarrow$ Fundamental mode in buncher cavity \Rightarrow speed modulation of electrons \Rightarrow bunched beam
- Bunched beam \Rightarrow excited fundamental mode in catcher cavity.
Kinetic energy \rightarrow EM energy
- Extract power from catcher cavity $\Rightarrow V_{out}$

The system again

