Nordic Particle Accelerator School

Rf exercises

1

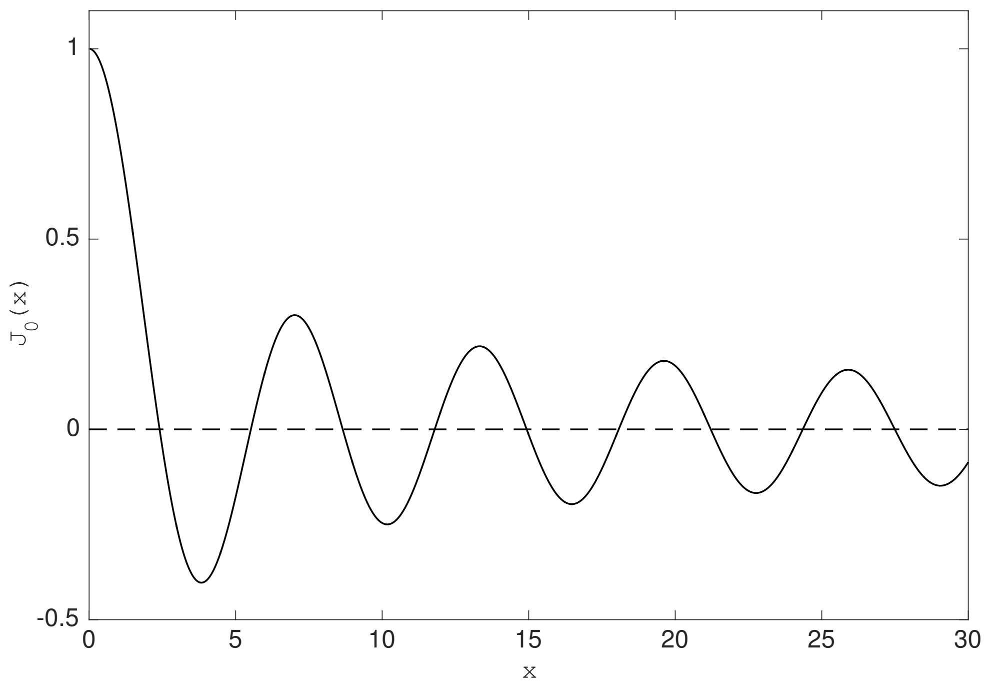

Figure 1: The besselfunction $J_0(x)$

The fundamental mode (resonance) in a pillbox cavity is called the TM010 mode. The electric field of that mode is given by

$$\boldsymbol{E}(\rho, t) = E_0 J_0(k\rho)\cos(\omega t)\hat{\boldsymbol{z}} \tag{1}$$

Here $\rho = \sqrt{x^2 + y^2}$ is the radial distance from the $z-$axis, E_0 is the amplitude and depends on how much energy that is stored in the cavity. The function $J_0(k\rho)$ is the Bessel function of order zero. The wavenumber $k = \frac{\omega}{c}$ is given by the boundary condition $J_0(ka) = 0$, where a is the radius of the cavity.

In the figure you can see a graph of $J_0(x)$. The first four zeros are at $x_1 = 2.405$, $x_2 = 5.520$, $x_3 = 8.654$, and $x_4 = 11.79$, Assume that you should design a cavity for an accelerator that uses waves with frequency 704 MHz.

a) Determine the smallest radius a such a cavity can have.

b) The cavity will have other resonance frequencies than 704 MHz. Determine three of these.

2

A klystron has 60 dB gain and its efficiency is 65%. The input power is 10 W.

a) What is the output power from the klystron?

b) How much power does the power source deliver to the klystron?

Hint: The efficiency η of an amplifier is defined by $\eta = \dfrac{\text{output power}}{\text{total input power}}$. The 10 W of the input signal can be neglected compared to the input power from the power source.

3

A klystron feeds power to a cavity in a linear accelerator. The klystron has 60.5 dB gain. When the waves travel through the waveguides, from the klystron to the cavity, they are attenuated 0.5 dB. Assume that the power of the input signal to the klystron is 10 W. What is the power delivered to the cavity?

Hint: You don't need a calculator. You can solve it with paper or pen, or even in your head.

4

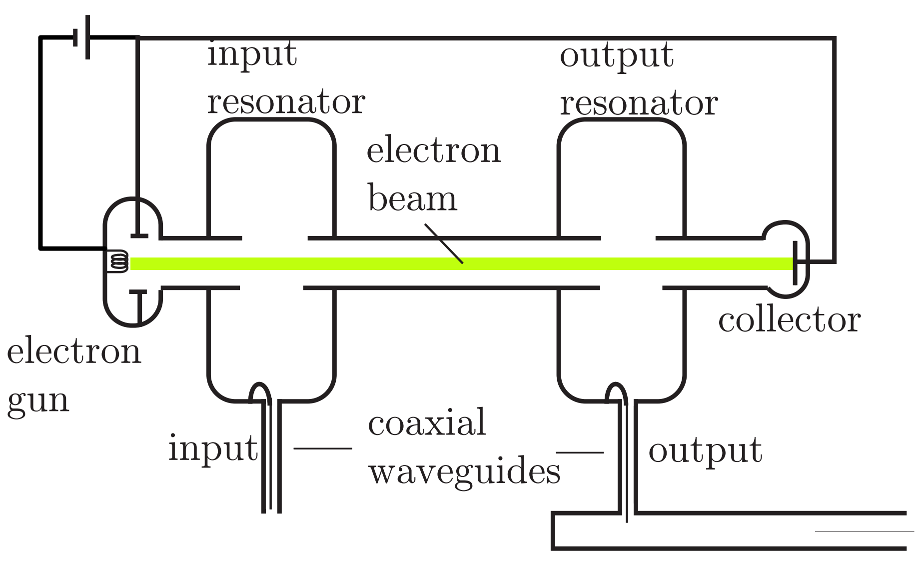

Figure 2: A klystron without input signal

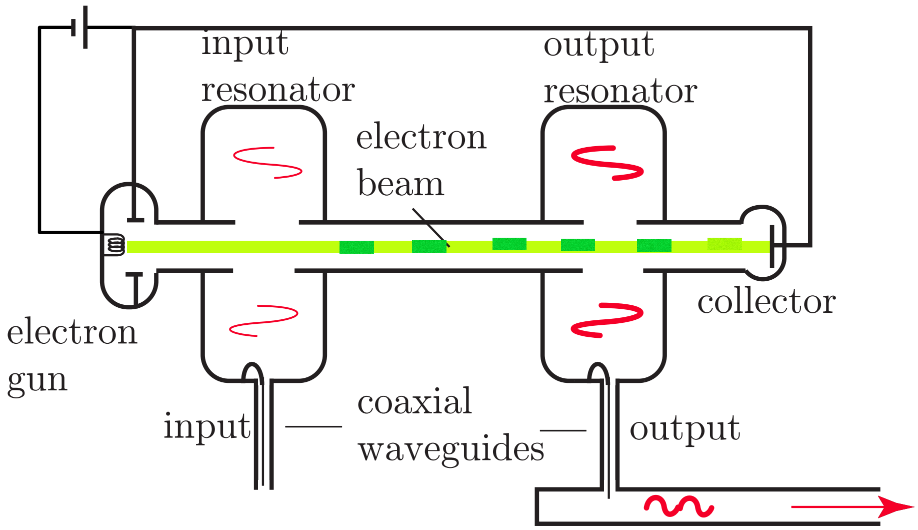

Figure 3: A klystron with input and output signals

In figures 2 and 3 you see a klystron. The electrons are generated at the filament in the electron gun. The input resonator (bucnher cavity) is fed by an input signal with the same frequency as the fundamental mode of the cavity. When the electron beam passes through the input resonator it is velocity modulated by the electric field of the fundamental mode. The modulation makes the electron beam bunched as it travels through the beam pipe. The bunched beam will then excite the fundamental mode in the output resonator (catcher cavity). The power from the output resonator is received by an antenna and then led down a coaxial waveguide.

a) What is the direction of the electric field in the catcher cavity when a bunch of electrons is inside the cavity?

b) Explain how the electron beam gets bunched.

5

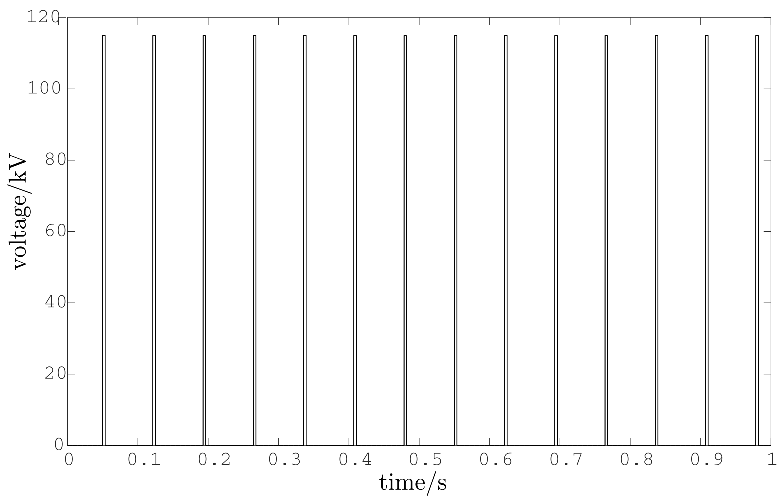

Figure 4: The output square pulse signal from the modulator for the 704 MHz klystrons at ESS.

The power source to a klystron is called modulator. These are often pulsed so that they deliver a square pulse signal of high voltage to the klystron. The modulators that feed the elliptical cavities at ESS deliver square pulses with pulse length 3.5 ms and a voltage of 115 kV, with a frequency of 14 Hz (14 pulses/second), see figure 4.

a) Determine the current of electrons in the klystron if the power leaving the modulator during a pulse is 11.5 MW.

b) The modulator gets a constant power from the power distribution system. That energy is stored in capacitors during the time interval when the voltage in Figure 4 is zero. Assume that we can neglect the power losses in the modulator. What is the power (time average) that it receives from the power distribution system.

6

A pill-box cavity is used for accelerating bunches of protons in a linear accelerator. The electric field along the symmetry axis of the cavity varies in time as in figure 5. Assume that the cavity is short enough so that it takes $T/30$ to travel through the cavity. Six different times are marked in the graph. At which of these is it best to let the bunch enter the cavity?

Hint: It is important that the bunches get elongated when they pass through a cavity.

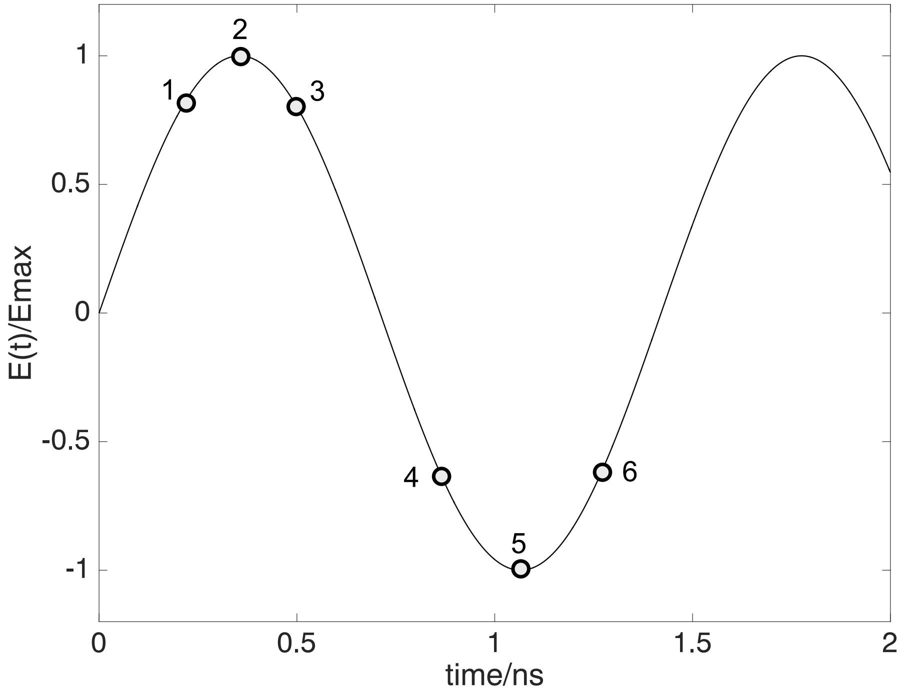

Figure 5: The amplitude of the electric field in the pill-box cavity. Positive values mean that the electric field is in the forward direction.

7

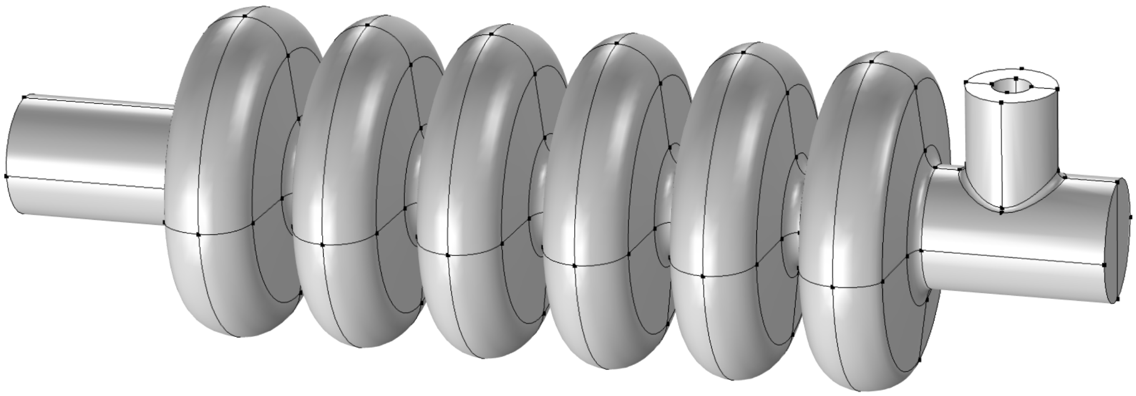

Figure 6: A 6-cell elliptical cavity

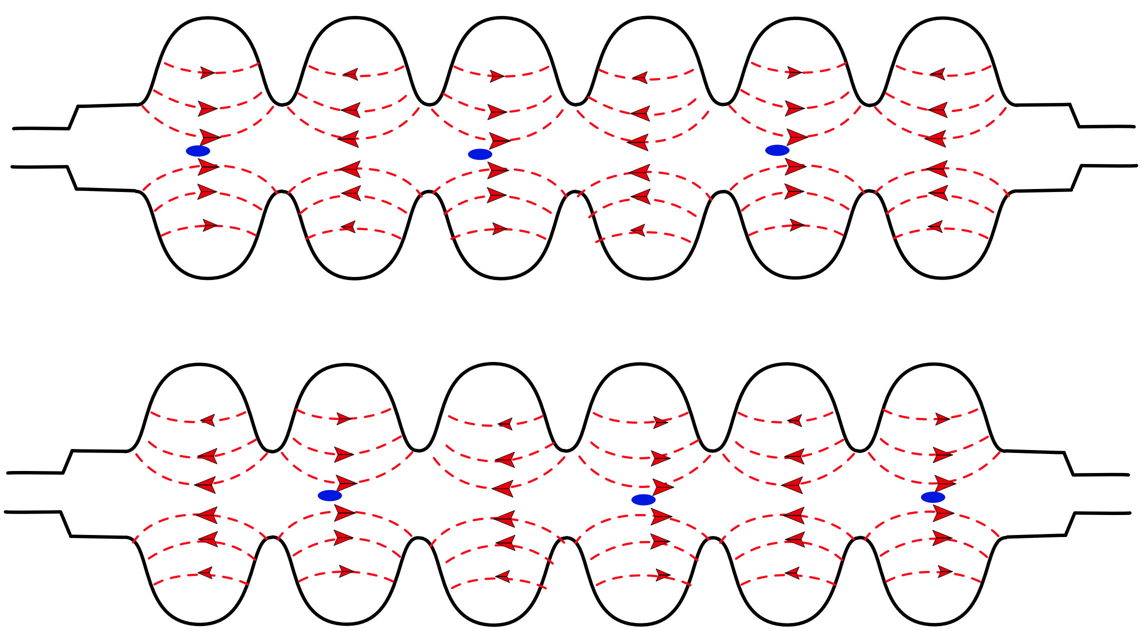

Figure 7: The electric field of the Pi-mode in a 6-cell elliptical cavity

In the figure you can see an elliptical cavity. These are used in the superconducting part of ESS, where they increase the energy of the protons from 250 MeV to 2 GeV. The cavity consists of six cells. There is a coupling between the cells so that the power that is put into the cavity is more or less evenly distributed among the cells. Just like other cavities there are an infinite number of resonances (modes) that can exist in the cavity. The one that is used for accelerating particles is called the pi-mode. It says that there is a phase shift of π radians between two neighbouring cells. In the figure you can see a snapshot of the electric field in the cavity. The blue dots mark bunches.

The elliptical cavities of ESS are fed with waves of frequency 704 MHz. Assume that the cavities are designed for protons that have speed $v = 0.6c$ when they travel through a cavity.

a) What is the distance d between the centers of two neighbouring cells?

b) In the design of the elliptical cavity there are a number of constraints that have to be fulfilled. One is that one should not have a resonance frequency that is close to $n \cdot 704$ MHz, where n is an integer larger than one. Why do they have this constraint?

8

An important parameter for a cavity is the quality factor. It tells us how much power that is lost in the cavity relative the stored energy in the cavity. The definition is

$$Q = 2\pi \frac{\text{stored electromagnetic energy in the cavity}}{\text{lost energy during one period}} \tag{2}$$

There are different phenomena that contribute to the lost energy. Assume that we have a normal conducting cavity with a power coupler and that we have a beam of particles traveling through the cavity.

a) Find three phenomena that can contribute to the losses.

b) Assume that the three phenomena give the lost energies P_1, P_2, and P_3 in the cavity. Show that

$$\frac{1}{Q} = \frac{1}{Q_1} + \frac{1}{Q_2} + \frac{1}{Q_3} \tag{3}$$

where Q_n is the quality factor that we would have if we only had the phenomena that gives rise to P_n.

9

In figure 9 you can follow a particle bunch that travels through a Drift tube linac (DTL). The DTL consists of a long cylindrical cavity with small cylindrical metallic tubes, called drift tubes, along the symmetry axis. The cylinder is filled with a resonance with the electric field seen in these figures. Without the drift tubes the electric field of this resonance is as in figure 9. The electric field without drift tubes is the same as we have seen earlier in the pill-box cavity, i.e.

$$\boldsymbol{E}(\rho, t) = E_0 J_0(k\rho) \cos \omega t \hat{\boldsymbol{z}} \tag{4}$$

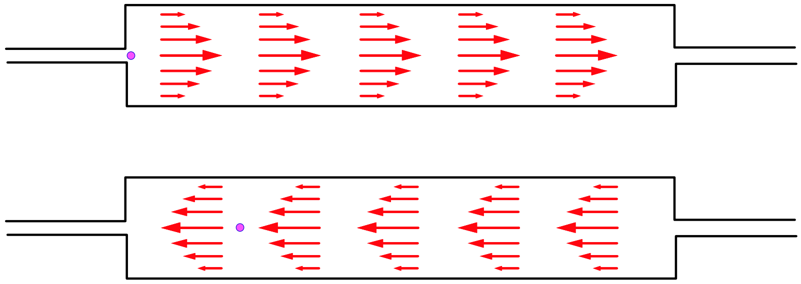

Figure 8: The electric field in a DTL without drift tubes at time $t = 0$ (upper) and at $t = T/2$ (lower). The red spot is a bunch of protons

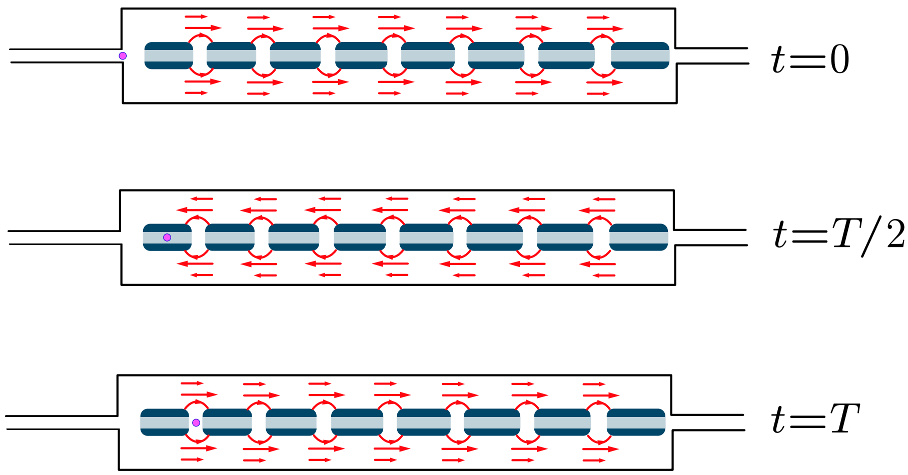

Figure 9: The electric field in a DTL with drift tubes at time $t = 0$, $t = T/2$ and $t = T$. The red spot is a bunch of protons

a) Why cannot we just use the cavity without drift tubes?

b) How can the problem in a) be solved by the drift tubes?

c) What is the length between the centers of two adjacent drift tubes if the frequency of the waves is 352 MHz and the bunch has the speed $v = 0.1c$ when it passes through these tubes?

d) Why do the lengths of the drift tubes slightly increase as we move to the right in the figure?

10

At a certain time t and a certain location z along a coaxial waveguide the electric field and the magnetic field are as in the figure.

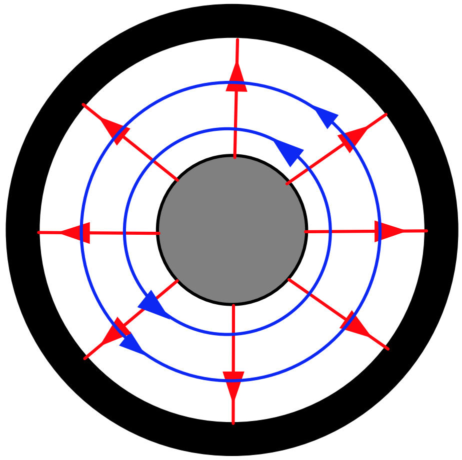

Figure 10: The electric field (red) and magnetic field (blue) in a coaxial waveguide

a) Is the wave traveling towards you or from you?

b) Are there any surface charges on the surfaces? If there are, mark positive surface charges with + and negative with -.

c) Are there any surface currents on the conductors? If there are, in what directions do these currents flow?

d) The currents in a conductor tend to flow close to the surface if they are varying in time. A good approximation is that the current is flowing in a thin layer. For time harmonic currents the thickness of the layer is called the skin depth and is given by

$$\delta = \sqrt{\frac{2}{\omega \mu_0 \sigma}} \tag{5}$$

where ω is the angular frequency, $\mu_0 = 4\pi \cdot 10^{-7}$ Vs/Am, and σ is the conductivity of the conductor. What is the skin depth in copper at a frequency of 704 MHz?